Exponential Growth & Decay

Growth and decay is proportional to population.

$$\frac{dP}{dt} = kP$$

$$P = Ae^{kt}$$

$P =$ population at time t

$k =$ growth(or decay) constant

$A =$ initial population

$t =$ time

Proof:

$$P = Ae^{kt}$$

$$\frac{dP}{dt} = kAe^{kt}$$

$$\frac{dP}{dt} = kP$$

e.g.(i) The growth rate per hour of a population of bacteria is 10% of the population. The initial population is 1000000

a) Show that $P = Ae^{0.1t}$ is a solution to the differential equation.

$$P = Ae^{0.1t}$$

$$\frac{dP}{dt} = 0.1Ae^{0.1t}$$

$$= 0.1P$$

b) Determine the population after $3\frac{1}{2}$ hours correct to 4 significant figures.

when $t = 0, P = 1000000$

$\therefore A = 1000000$

$P = 1000000e^{0.1t}$

when $t = 3.5, P = 1000000e^{0.1(3.5)}$

$= 1419000$

$\therefore$ after $3\frac{1}{2}$ hours there is 1419000 bacteria

(ii) On an island, the population in 1960 was 1732 and in 1970 it was 1260.

a) Find the annual growth rate to the nearest %, assuming it is proportional to population.

$$\frac{dP}{dt} = kP$$

$$P = Ae^{kt}$$

when $t = 0, P = 1732$

$$\therefore A = 1732$$

$$P = 1732e^{kt}$$

when $t = 10, P = 1260$

$$\text{i.e.}\ 1260 = 1732e^{10k}$$

$$e^{10k} = \frac{1260}{1732}$$

$$10k = \log\left(\frac{1260}{1732}\right)$$

$$k = \frac{1}{10}\log\left(\frac{1260}{1732}\right)$$

$$k = -0.0318165$$

$\therefore$ growth rate is - 3%

b) In how many years will the population be half that in 1960?

$$\text{when } P = 866, \quad 866 = 1732e^{kt}$$

$$e^{kt} = \frac{1}{2}$$

$$kt = \log\frac{1}{2}$$

$$t = \frac{1}{k}\log\frac{1}{2}$$

$$t = 21.786$$

$\therefore$ In 22 years the population has halved

Exercise 16B; 4, 5, 7, 8, 10, 11, 13