General Solutions of Trig Equations
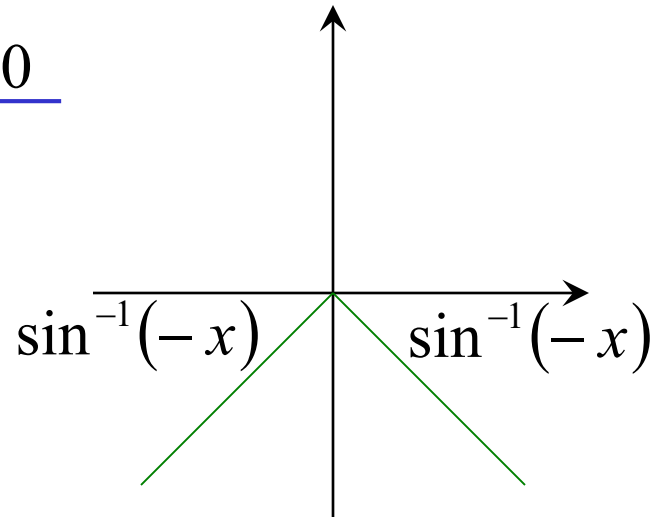
$\sin\theta = x$

$x > 0$

$$\sin^{-1}x \quad \sin^{-1}x$$

$\theta = \sin^{-1}x \text{ or } \pi - \sin^{-1}x$

$x < 0$
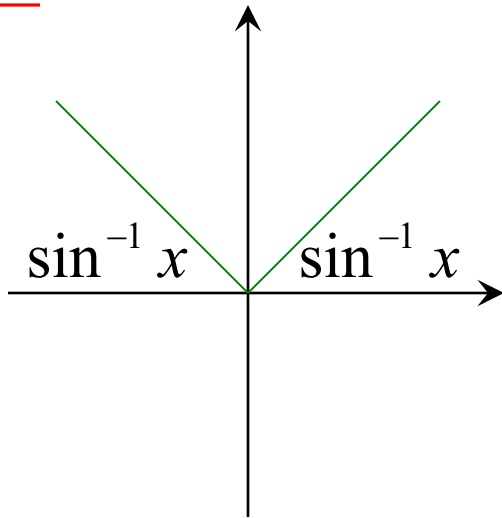
$$\sin^{-1}(-x) \quad \sin^{-1}(-x)$$
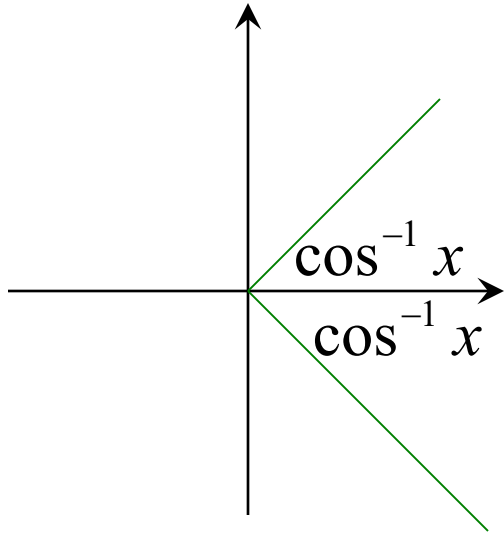
$\theta = \pi + \sin^{-1}(-x) \text{ or } 2\pi - \sin^{-1}(-x)$

$\theta = \pi - \sin^{-1}x \text{ or } 2\pi + \sin^{-1}x$

$\sin\theta = x$

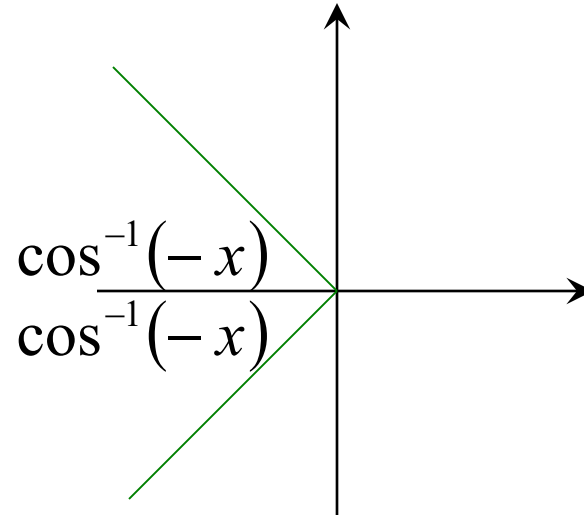$\theta = \pi k + (-1)^k \sin^{-1}x \quad \text{where } k \in \mathbb{Z}$

$\underline{\cos\theta = x}$

$\underline{x > 0}$

$\underline{x < 0}$

$\theta = \cos^{-1} x \text{ or } 2\pi - \cos^{-1} x$

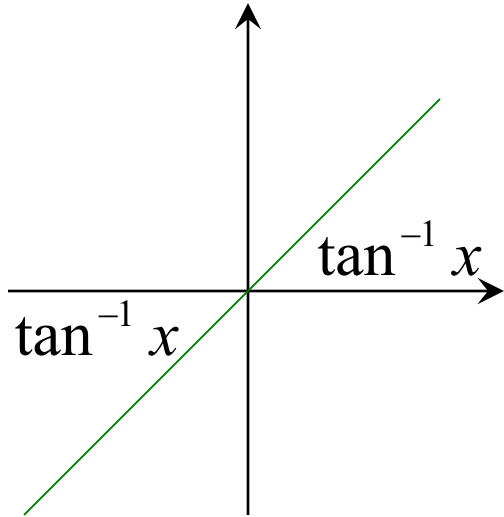$\theta = \pi - \cos^{-1}(-x) \text{ or } \pi + \cos^{-1}(-x)$

$\theta = \pi - (\pi - \cos^{-1} x) \text{ or } \pi + (\pi - \cos^{-1} x)$

$= \cos^{-1} x \qquad \text{or } 2\pi - \cos^{-1} x$

$$\cos\theta = x$$
$$\theta = 2\pi k \pm \cos^{-1} x \qquad \text{where } k \in \mathbb{Z}$$

$\underline{\tan\theta = x}$

$\underline{x > 0}$ $\underline{x < 0}$
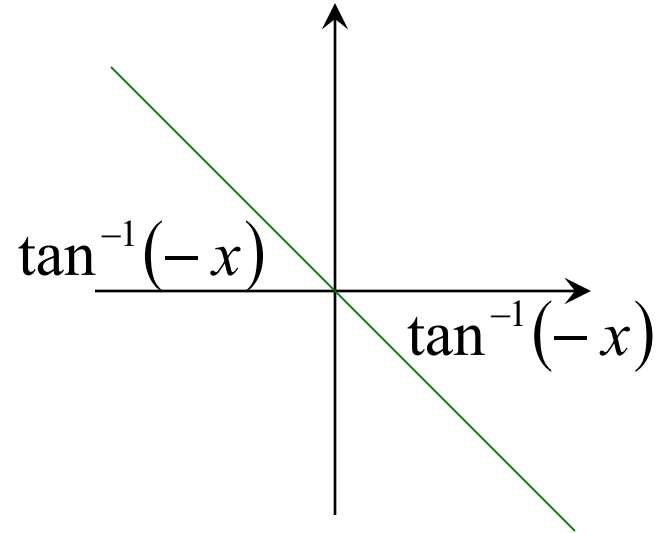
$\theta = \tan^{-1} x \text{ or } \pi + \tan^{-1} x$ $\theta = \pi - \tan^{-1}(-x) \text{ or } 2\pi - \tan^{-1}(-x)$

$\theta = \pi + \tan^{-1} x \text{ or } 2\pi + \tan^{-1} x$

$$\tan\theta = x$$
$$\theta = \pi k + \tan^{-1} x \qquad \text{where } k \in \mathbb{Z}$$

$$\text{e.g. } (i)\ \sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \pi k + (-1)^k \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

where $k \in \mathbb{Z}$

$$\theta = \pi k + (-1)^k \frac{\pi}{3}$$

OR

$$\sin\theta = \frac{\sqrt{3}}{2}$$

Q1, Q2

$$\theta = \frac{\pi}{3} + 2\pi k \ \text{ or } \ \frac{2\pi}{3} + 2\pi k$$

where $k \in \mathbb{Z}$

$$(ii)\ \cos\theta = -\frac{1}{\sqrt{2}}$$

$$\theta = 2\pi k \pm \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

where $k \in \mathbb{Z}$

$$\theta = 2\pi k \pm \frac{3\pi}{4}$$

OR

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

Q2, Q3

$$\theta = \frac{3\pi}{4} + 2\pi k \ \text{ or } \ \frac{5\pi}{4} + 2\pi k$$

where $k \in \mathbb{Z}$

$(iii)\tan\theta=\dfrac{1}{\sqrt{3}}$

$\theta=\pi k+\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

where $k\in\mathbb{Z}$

$\theta=\pi k+\dfrac{\pi}{6}$

OR

$\tan\theta=\dfrac{1}{\sqrt{3}}$

Q1

$\theta=\dfrac{\pi}{6}+\pi k$ where $k\in\mathbb{Z}$

$(iv)\sin\theta=\sin\dfrac{5\pi}{7}$

$\theta=\pi k+(-1)^{k}\sin^{-1}\sin\dfrac{5\pi}{7}$

$\theta=\pi k+(-1)^{k}\dfrac{2\pi}{7}$

where $k\in\mathbb{Z}$

$(v)\cos 2x=\cos\dfrac{\pi}{9}$

$2x=2\pi k\pm\cos^{-1}\cos\dfrac{\pi}{9}$

$2x=2\pi k\pm\dfrac{\pi}{9}$

$x=\pi k\pm\dfrac{\pi}{18}$ where $k\in\mathbb{Z}$

(vi) $\cos 4x = \cos x$

$$4x = 2\pi k \pm \cos^{-1} \cos x \quad \text{where } k \in \mathbb{Z}$$

if $x > 0$; $4x = 2\pi k \pm x$ if $x < 0$; $4x = 2\pi k \pm (\pi - x)$

$$4x \mp x = 2\pi k$$

$$4x = 2\pi k \pm (x - 2\pi)$$

$$4x \mp x = 2\pi k \mp 2\pi$$

$$3x = 2\pi k \quad \text{or} \quad 5x = 2\pi k$$

$$= 2\pi(k - 1) \text{ or } 2\pi(k + 1)$$

$$x = \frac{2\pi k}{3} \qquad x = \frac{2\pi k}{5}$$

$$\therefore x = \frac{2\pi k}{3} \text{ or } \frac{2\pi k}{5}, \text{ where } k \in \mathbb{Z}$$

RHS still generates multiples of 2π so there is no need to consider cases

(vii) $\sin 4x = \sin x$

$$4x = \pi k + (-1)^k \sin^{-1} \sin x \qquad \text{where } k \in \mathbb{Z}$$

if $x > 0$; $4x = \pi k + (-1)^k x$

<div style="border: 2px solid red; border-radius: 10px;">
no need to check the positive and negative cases for inverse sine
</div>

if k is odd ;

$$4x = \pi k - x$$

$$5x = \pi k$$

$$x = \frac{\pi k}{5}$$

$$x = \frac{\pi(2n+1)}{5} \quad n \in \mathbb{Z}$$

if k is even ;

$$4x = \pi k + x$$

$$3x = \pi k$$

$$x = \frac{\pi k}{3}$$

$$x = \frac{2\pi n}{3} \quad n \in \mathbb{Z}$$

if $x < 0$; $\sin^{-1} \sin x = x$ i.e. the same results as when $x > 0$

$$\therefore x = \frac{\pi(2k+1)}{5} \text{ or } \frac{2\pi k}{3} , \text{ where } k \in \mathbb{Z}$$

general solutions handout;
1, 4bdf, 5ace, 6acd, 7acehjk,
8ace, 9, 10, 11, 12ad, 13, 14