Graphing Inverse Trig Functions

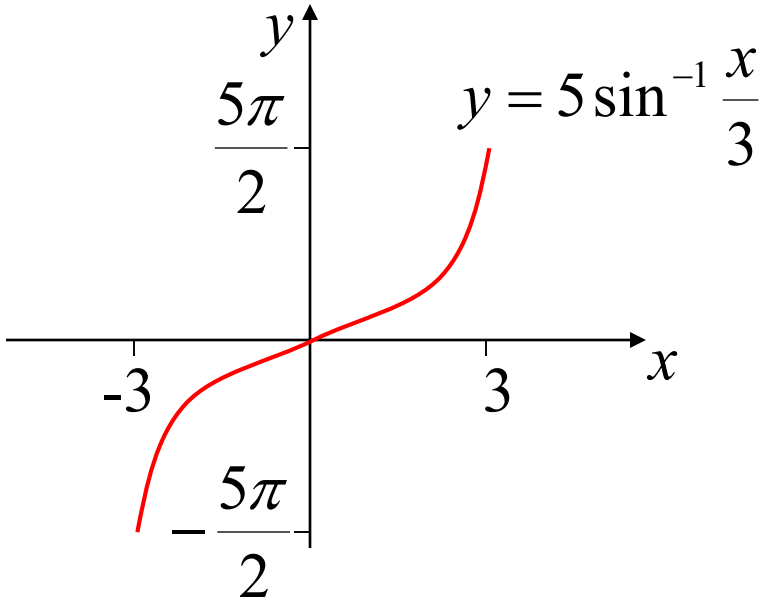
e.g $(i) y = 5\sin^{-1}\frac{x}{3}$

Domain: $-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$

Range: $-\frac{\pi}{2} \le \frac{y}{5} \le \frac{\pi}{2}$

$-\frac{5\pi}{2} \le y \le \frac{5\pi}{2}$



$(ii) y = \tan^{-1}\left(\sqrt{3-x^2}\right)$

Domain: $3 - x^2 \geq 0$

$$-\sqrt{3} \leq x \leq \sqrt{3}$$

Range: $x = \sqrt{3}, y = \tan^{-1} 0$

$$= 0$$

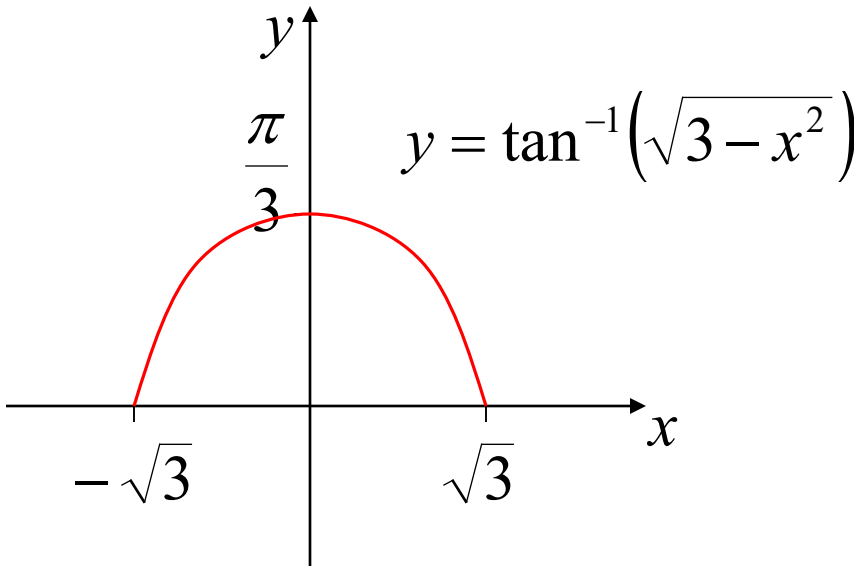
$$x = -\sqrt{3}, y = \tan^{-1} 0$$

$$= 0$$

$$x = 0, y = \tan^{-1} \sqrt{3}$$

$$= \frac{\pi}{3}$$

$$0 \leq y \leq \frac{\pi}{3}$$

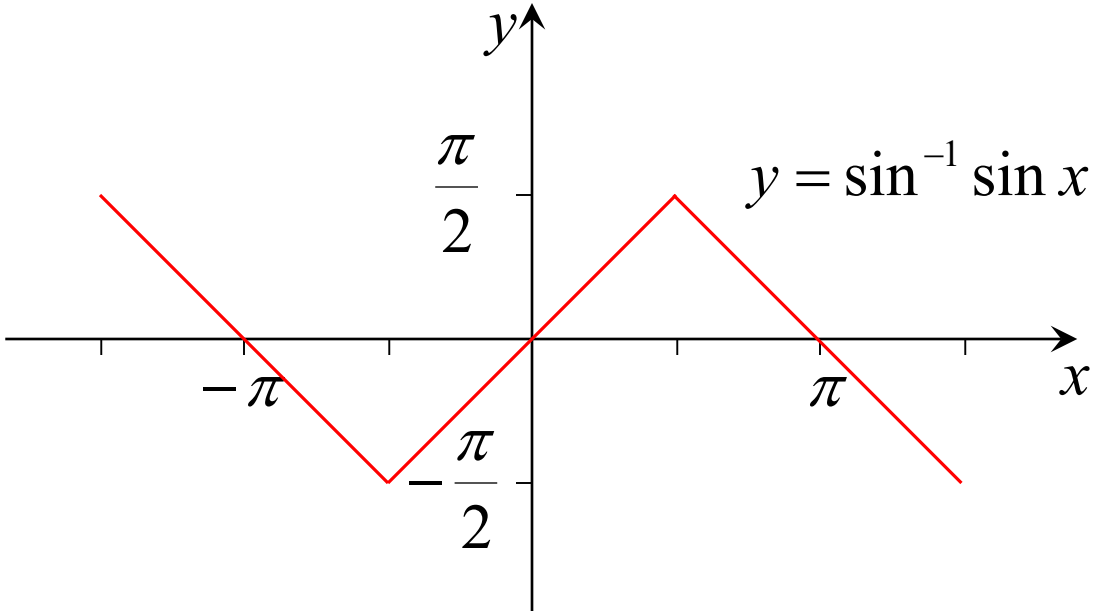


$(iii)\, y = \sin^{-1} \sin x$

Domain: $-1 \le \sin x \le 1$

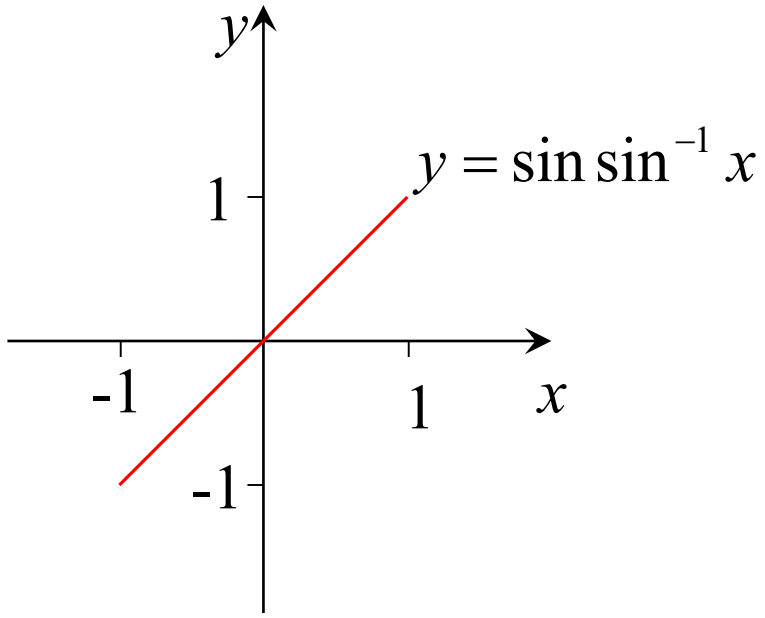
all real x

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



$(iv)\ y = \sin\sin^{-1} x$

Domain: $-1 \le x \le 1$



Range: when $x = 1, y = \sin\sin^{-1} 1$

$$= \sin\frac{\pi}{2}$$

$$= 1$$

when $x = -1, y = \sin\sin^{-1}(-1)$

$$= \sin\left(-\frac{\pi}{2}\right)$$

$$= -1$$

when $x = 0, y = \sin\sin^{-1} 0$

$$= \sin 0$$

$$= 0$$

$$-1 \le y \le 1$$

Exercise 17C; 2 to 6, 8 to 10