Sum To Infinity (Limiting Sum)

$\underline{NOTE}: |r| < 1$

If $|r| < 1$, $\quad \lim_{n \to \infty} r^n = 0$

The sequence converges to zero

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{a}{1 - r}$$

$$S_\infty = \frac{a}{1 - r}, \text{ if } |r| < 1$$

e.g. (***i***) Does $56 + 4 + \frac{2}{7} + \ldots$ have a limiting sum?

$$r = \frac{4}{56} < 1$$

$\therefore$ as $|r| < 1$, it has a limiting sum

$(ii)\ \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$$a=\frac{1}{2}, r=\frac{1}{2}$$

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{2}}$$

$$=1$$

(iii) Write $0.\dot{3}\dot{6}$ as a fraction

$$0.\dot{3}\dot{6}=0.36+0.0036+0.000036+\ldots$$

$$a=0.36, r=0.01$$

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{0.36}{1-0.01}$$

$$=\frac{36}{99}$$

$$=\frac{4}{11}$$

Exercise 1H; 5, 9, 10dh, 11a, 12ac, 13ab (i), 14bd, 15, 16ab, 18a,

Exercise 1I; 2eg, 3d, 4a