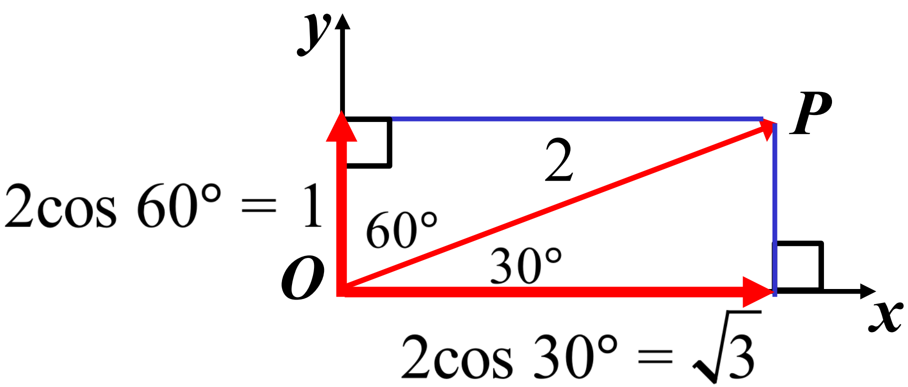
Vector Projections

When a vector is broken into components, it is rewritten as the projection of the vector onto the *x*-axis and the *y*-axis

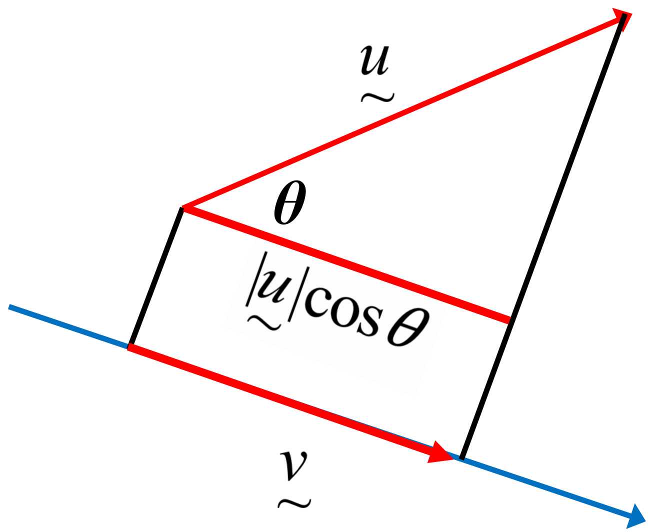


horizontal component of $\overrightarrow{OP} = \sqrt{3}\,\underset{\sim}{i}$

vertical component of $\overrightarrow{OP} = \underset{\sim}{j}$

$\overrightarrow{OP} = \sqrt{3}\,\underset{\sim}{i} + \underset{\sim}{j}$

horizontal and vertical was chosen for convenience, a vector can be projected onto any other vector



To project $\underset{\sim}{u}$ onto the vector $\underset{\sim}{v}$

* Drop perpendiculars from the endpoints of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$

* Calculate the length of the "shadow" of the projection

* Multiply by the unit vector in the direction of $\underset{\sim}{v}$

$$\text{projection of } \underset{\sim}{u} \text{ on } \underset{\sim}{v} = (|\underset{\sim}{u}|\cos\theta)\,\hat{\underset{\sim}{v}}$$

Now $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$

$$|\underset{\sim}{u}|\cos\theta = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|}$$

$$\text{proj}_{\underset{\sim}{v}}\,\underset{\sim}{u} = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|} \times \hat{\underset{\sim}{v}}$$

$$= \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|} \times \frac{\underset{\sim}{v}}{|\underset{\sim}{v}|}$$

$$= \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|^2} \times \underset{\sim}{v}$$

$$\text{proj}_{\underset{\sim}{v}}\,\underset{\sim}{u} = \left(\frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\underset{\sim}{v} \cdot \underset{\sim}{v}}\right)\underset{\sim}{v}$$

Notes: $\text{proj}_{\underset{\sim}{v}}\,\lambda\underset{\sim}{u} = \lambda\,\text{proj}_{\underset{\sim}{v}}\,\underset{\sim}{u}$

$\text{proj}_{\underset{\sim}{w}}\,(\underset{\sim}{u} + \underset{\sim}{v}) = \text{proj}_{\underset{\sim}{w}}\,\underset{\sim}{u} + \text{proj}_{\underset{\sim}{w}}\,\underset{\sim}{v}$

dot product and division are **NOT** inverse operations, so you cannot cancel e.g. (i) Find the length of the projection of $\underset{\sim}{a} = 5\underset{\sim}{i} - \underset{\sim}{j}$ onto $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$$\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|} = \frac{(5)(3) + (-1)(4)}{\sqrt{3^2 + 4^2}}$$

$$= \frac{11}{5}$$

the length of the projection is also known as the **scalar projection**

(ii) Find the projection of $\underset{\sim}{u} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ onto $\underset{\sim}{v} = -2\underset{\sim}{i} + 3\underset{\sim}{j}$

$$\text{proj}_{\underset{\sim}{v}} \underset{\sim}{u} = \frac{(2)(-2) + (-5)(3)}{(-2)^2 + 3^2} \times \left(-2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$$

$$= -\frac{19}{13}\left(-2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$$

$$= \frac{38}{13}\underset{\sim}{i} - \frac{57}{13}\underset{\sim}{j}$$

Exercise 8E;
1a, 2b, 3a, 4a, 5, 6ac, 7b, 8b, 9, 10, 11, 13, 14