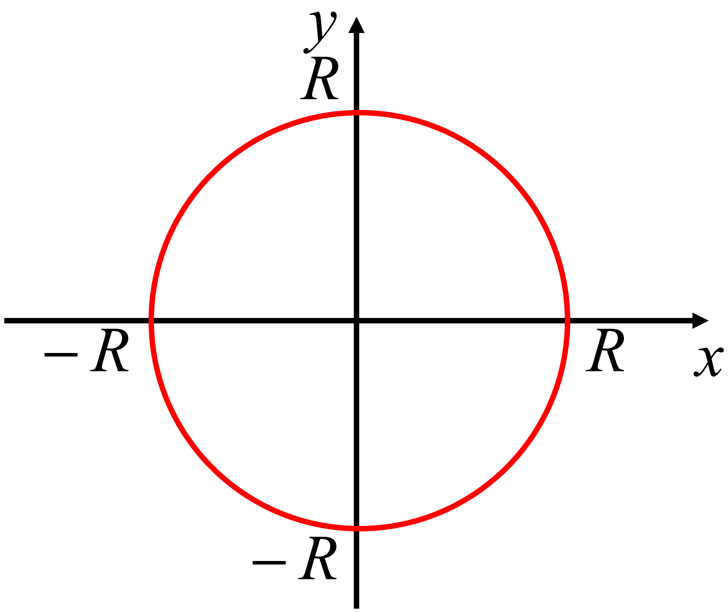
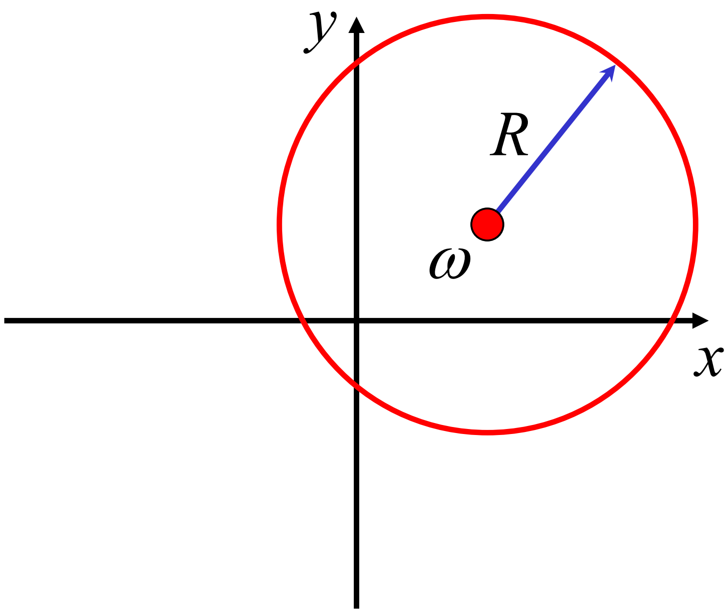
Locus and Complex Numbers

Circles





$$z\bar{z} = R^2$$

or

$$|z| = R$$

$$(z-\omega)(\bar{z}-\overline{\omega}) = R^2$$

or

$$|z-\omega| = R$$

e.g. (i) Express these circles in terms of z

a) $x^2 + y^2 = 16$

$$|z| = 4$$

$$(z\bar{z} = 16)$$

b) $x^2 + y^2 + 6x - 4y - 12 = 0$

$$x^2 + 6x + y^2 - 4y = 12$$

$$(x+3)^2 + (y-2)^2 = 25$$

$$|z + 3 - 2i| = 5$$

$$[(z + 3 - 2i)(\bar{z} + 3 + 2i) = 25]$$

(ii) Find the centre and radius of;

a) $|z - 5 - i| = 2$

centre : $(5,1)$

radius : 2 units

b) $(z + 4 + i)(\bar{z} + 4 - i) = 49$

centre : $(-4,-1)$

radius : 7 units

c) $|3z| = |z+2-i|$

$$3|z| = |z+2-i|$$

$$9x^2 + 9y^2 = (x+2)^2 + (y-1)^2$$

$$9x^2 + 9y^2 = x^2 + 4x + 4 + y^2 - 2y + 1$$

$$8x^2 - 4x + 8y^2 + 2y = 5$$

$$x^2 - \frac{1}{2}x + y^2 + \frac{1}{4}y = \frac{5}{8}$$

$$\left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{8}\right)^2 = \frac{45}{64}$$

$$\text{centre}: \left(\frac{1}{4}, -\frac{1}{8}\right)$$

$$\text{radius}: \frac{3\sqrt{5}}{8} \text{ units}$$

d) $z\bar{z} + 2(z + \bar{z}) = 0$

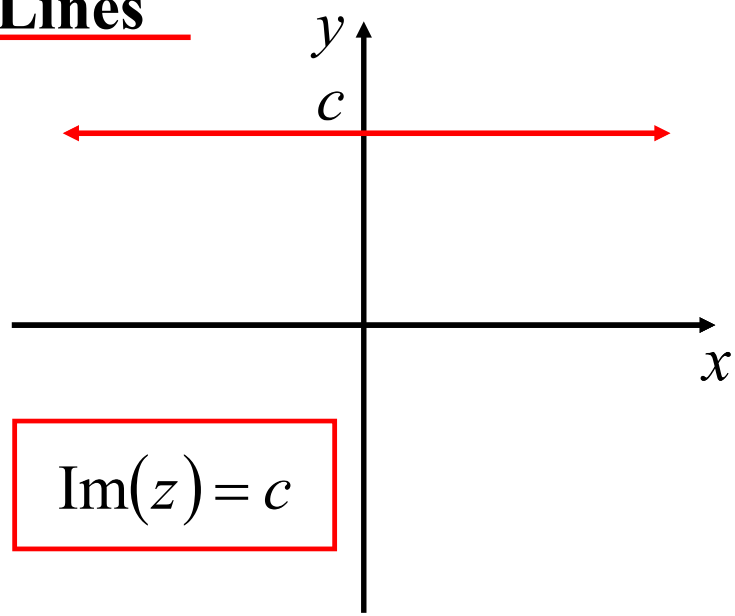
$$x^2 + y^2 + 4x = 0$$

$$(x+2)^2 + y^2 = 4$$

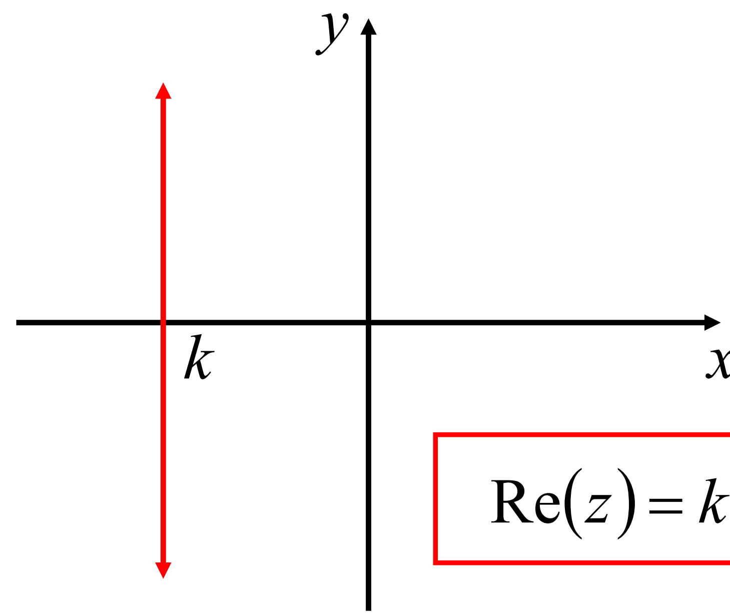
$$\text{centre}: (-2, 0)$$

$$\text{radius}: 2 \text{ units}$$

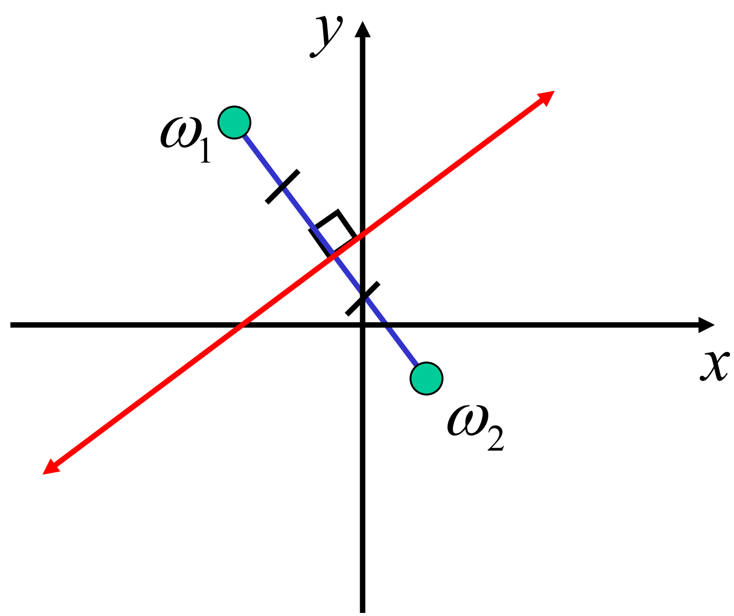
Lines



$$\mathrm{Im}(z) = c$$



$$\mathrm{Re}(z) = k$$



$$|z - \omega_1| = |z - \omega_2|$$

$e.g.\ |z-1-i| = |z+2+i|$

$$(x-1)^2 + (y-1)^2 = (x+2)^2 + (y+1)^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = x^2 + 4x + 4 + y^2 + 2y + 1$$

$$\underline{6x + 4y + 3 = 0}$$

$OR\ \perp$ bisector of $(1,1)$ and $(-2,-1)$

$$M = \left(\frac{1-2}{2}, \frac{1-1}{2}\right) \qquad m = \frac{1+1}{1+2}$$

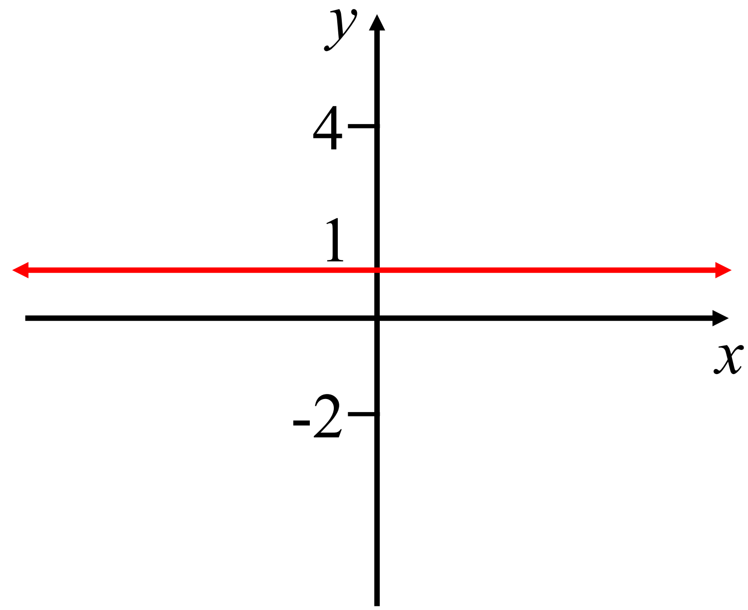
$$= \left(-\frac{1}{2}, 0\right) \qquad = \frac{2}{3} \qquad \therefore \text{ required slope is } -\frac{3}{2}$$

$$y - 0 = -\frac{3}{2}\left(x + \frac{1}{2}\right)$$

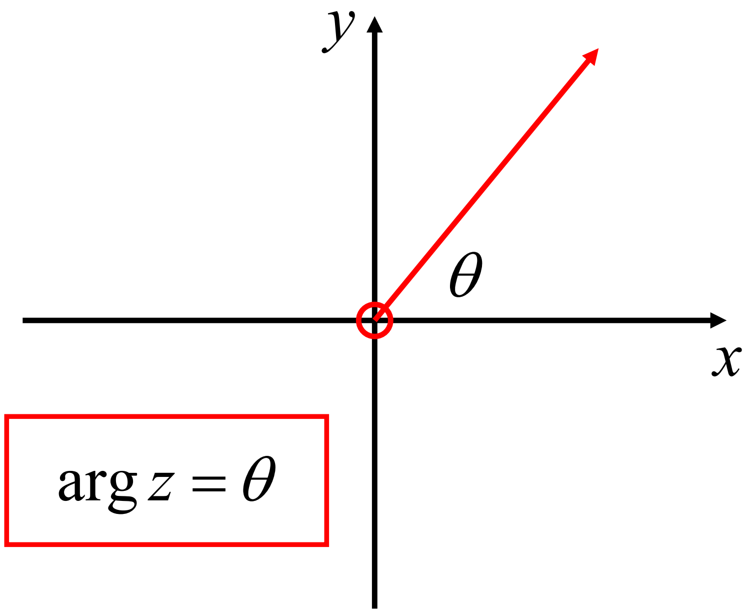
$$2y = -3x - \frac{3}{2}$$

$$\underline{6x + 4y + 3 = 0}$$

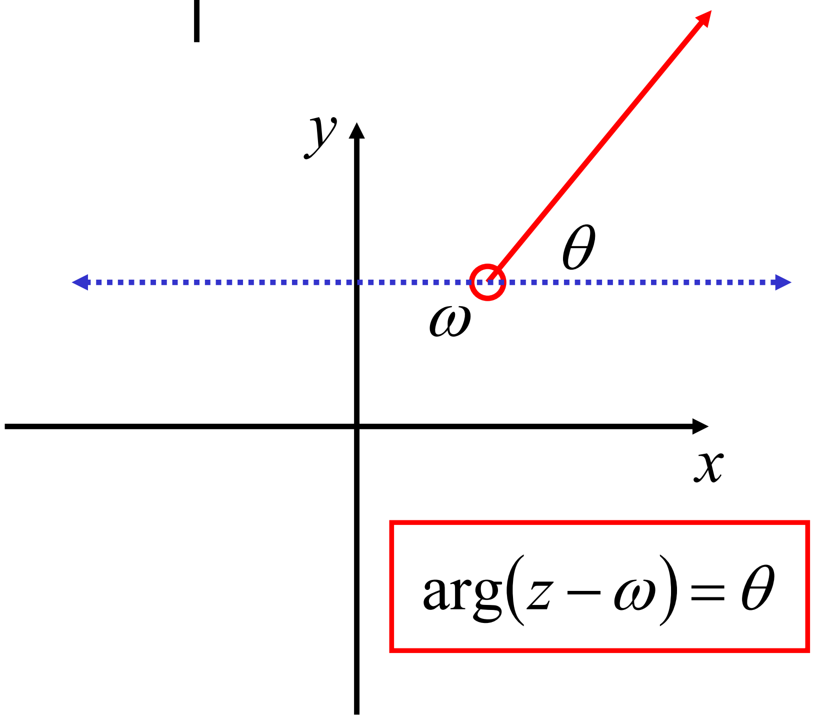
(ii) Sketch $|z+2i| = |z-4i|$



Rays

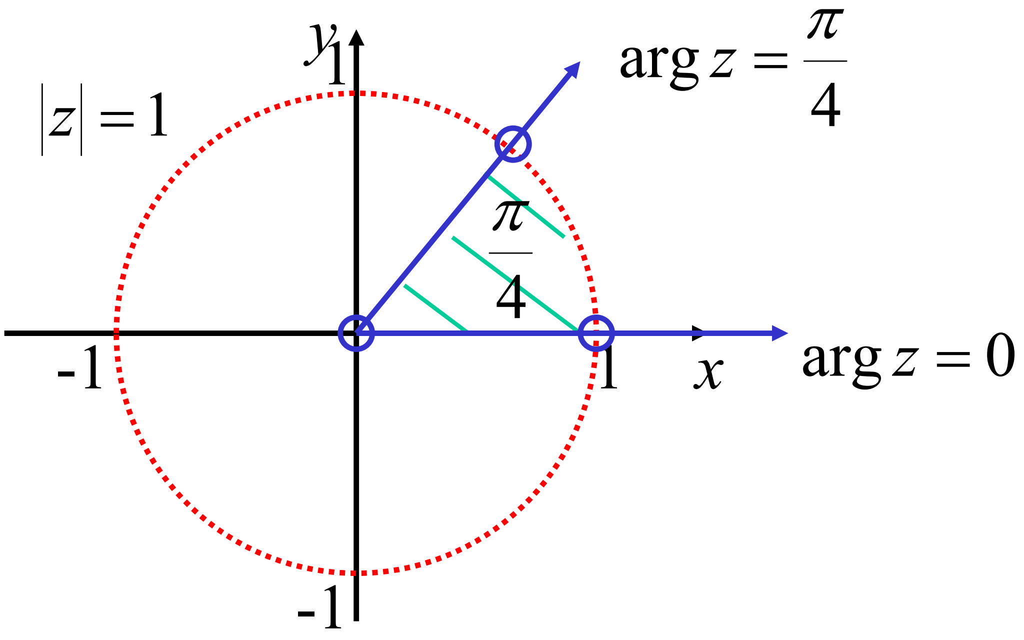


$$\arg z = \theta$$

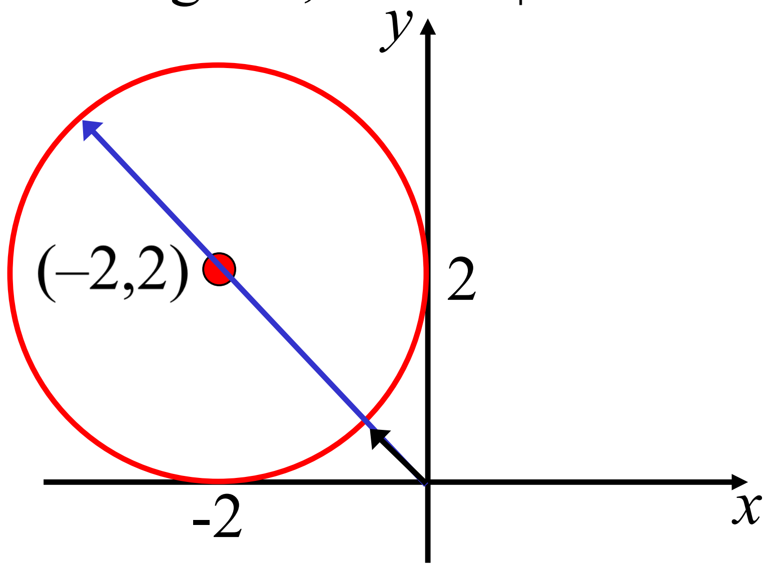


$$\arg(z-\omega) = \theta$$

e.g. $|z| < 1$ and $0 \le \arg z \le \dfrac{\pi}{4}$



e.g. (i) On an Argand diagram, sketch $|z + 2 - 2i| = 2$



Exercise 1F;
1 to 5 ace etc, 6,
7 ac, 10, 11, 12a, 13

***Patel:* Exercise 4M;**
1ac, 2bd, 3ac,
4bdf, 5bd, 6ac

***Patel:* Exercise 4N;**
1acfhj, 2ace,
3acegikl, 4ace

(ii) Find the possible values of arg z

The tangents drawn from the origin to the circle will create the vectors with the minimum and maximum arguments

$$\frac{\pi}{2} \leq \arg z \leq \pi$$

(iii) Find the minimum and maximum values of $|z|$

Draw a secant that goes through the origin and the centre of the circle, this will create the vectors with minimum and maximum modulus.

distance to centre = $2\sqrt{2}$

radius = 2

$\min |z| = 2\sqrt{2} - 2$

$\max |z| = 2\sqrt{2} + 2$