Curve Sketching & Calculus

<u>The First Derivative</u> $y', f'(x), \dfrac{d}{dx}\{f(x)\}, \dfrac{dy}{dx}$

$\dfrac{dy}{dx}$ measures the slope of the tangent to a curve

If $f'(x) > 0$, the curve is increasing

If $f'(x) < 0$, the curve is decreasing

If $f'(x) = 0$, the curve is stationary

e.g. For the curve $y = 3x^2 - x^3$, find all of the stationary points

and determine their nature.

Hence sketch the curve

$$\frac{dy}{dx} = 6x - 3x^2$$

Stationary points occur when $\dfrac{dy}{dx} = 0$

i.e. $6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0 \quad \text{or} \quad x = 2$

$\therefore$ stationary points occur at $(0,0)$ and $(2,4)$

$(0,0)$

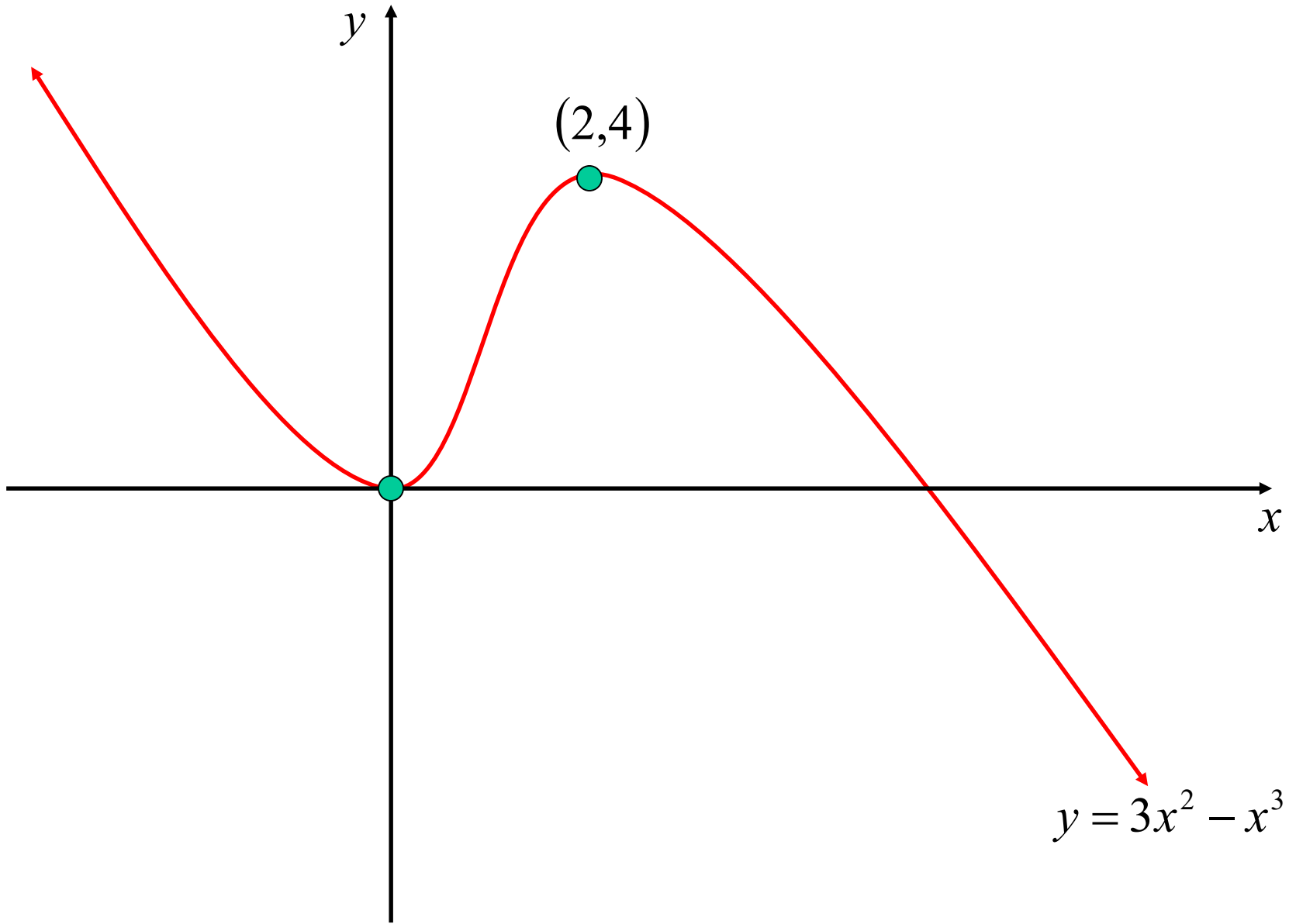| x	$0^{-}_{(-1)}$	0	$0^{+}_{(1)}$
$\dfrac{dy}{dx}$	(-9) ╲	0 —	(3) ╱

$\therefore (0,0)$ is a minimum turning point

$(2,4)$

x	$2^{-}_{(1)}$	2	$2^{+}_{(3)}$
$\dfrac{dy}{dx}$	(3) ╱	0 —	(-9) ╲

$\therefore (2,4)$ is a maximum turning point

y

$(2,4)$

x

$y = 3x^2 - x^3$

Exercise 4A; 1, 2ace, 4, 5, 6ac, 7, 8, 11, 12ace etc, 13, 15

Exercise 4B; 3ad, 6, 8ac, 9, 11, 14 to 17, 19