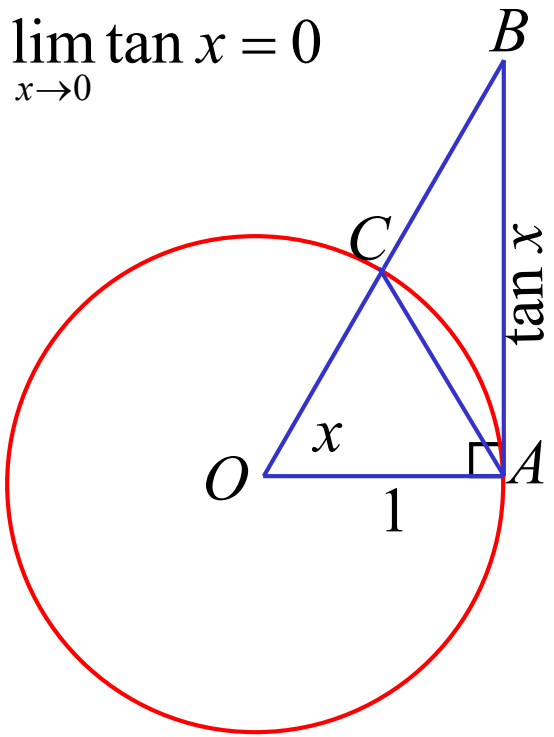
Differentiating Trig

$\lim_{x\to 0} \sin x = 0$

$\lim_{x\to 0} \cos x = 1$

$\lim_{x\to 0} \tan x = 0$



Area $\Delta AOC <$ Area Sector $OAC <$ Area ΔAOB

$$\frac{1}{2}(1)(1)\sin x < \frac{1}{2}(1)^2 x < \frac{1}{2}(1)\tan x$$

$$\sin x < x < \tan x$$

$$\frac{\sin x}{\sin x} < \frac{x}{\sin x} < \frac{\tan x}{\sin x}$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

as $x \to 0$

$$1 < \frac{x}{\sin x} < 1$$

$$\lim_{x\to 0} \frac{x}{\sin x} = 1$$

e.g. $(i)\lim_{x\to 0}\dfrac{5x}{\sin 5x} = 1$

$(ii)\lim_{x\to 0}\dfrac{x}{\sin 3x} = \lim_{x\to 0}\dfrac{1}{3}\times\dfrac{3x}{\sin 3x}$

$= \dfrac{1}{3}$

$y = \sin x$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sin h}{h}\right) + \sin x\left(\frac{\cos h - 1}{h}\right)$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sin h}{h}\right) + \sin x\left(\frac{2\cos^2\left(\frac{h}{2}\right) - 2}{h}\right) \qquad \left[\cos 2\theta = 2\cos^2\theta - 1\right]$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sin h}{h}\right) - \sin x\left(\frac{2\sin^2\left(\frac{h}{2}\right)}{h}\right)$$

$$= \lim_{h \to 0} \cos x \left(\frac{\sin h}{h} \right) - \sin x \left(\frac{\sin^2 \left(\frac{h}{2} \right)}{\frac{h}{2}} \right)$$

$$= \lim_{h \to 0} \cos x \left(\frac{\sin h}{h} \right) - \sin x \left(\frac{\sin \left(\frac{h}{2} \right)}{\frac{h}{2}} \times \sin \left(\frac{h}{2} \right) \right)$$

$\sin h \to 1$, $\frac{\sin \left(\frac{h}{2} \right)}{\frac{h}{2}} \to 1$, $\sin \left(\frac{h}{2} \right) \to 0$

$$= (\cos x)(1) + (\sin x)(0)$$

$$= \cos x$$

$$y = \cos x$$

$$y = \sin \left(\frac{\pi}{2} - x \right)$$

$$\frac{dy}{dx} = -\cos \left(\frac{\pi}{2} - x \right)$$

$$= -\sin x$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$y = \tan x$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x)\sec^2 f(x)$$

e.g. $(i)\ y = \sin x^3$

$$\frac{dy}{dx} = 3x^2 \cos x^3$$

$(ii)\ y = \tan \frac{1}{x}$

$$\frac{dy}{dx} = -\frac{1}{x^2}\sec^2 \frac{1}{x}$$

$(iii)\ y = \log \cos x$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

$(iv)\ y = \tan^5 x$

$$\frac{dy}{dx} = 5\tan^4 x \sec^2 x$$

$(v)\ y = \cos e^x$

$$\frac{dy}{dx} = -e^x \sin e^x$$

Exercise 7A; 6, 7, 8, 11, 15abc

Exercise 7B; 1denp, 2achjk, 6a, 7ac, 8ade, 11bd, 12cdf, 13ac, 16b, 10b, 19, 20, 21, 23

Exercise 7C; 1e, 4c, 7, 9, 10, 13, 14, 15, 17, 21