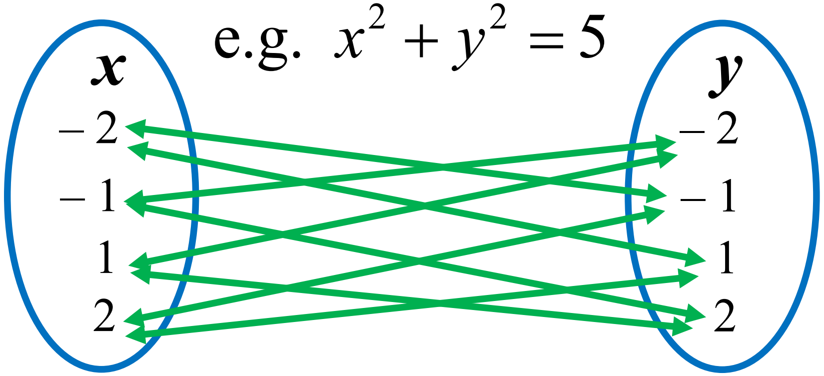
Functions

Definitions:

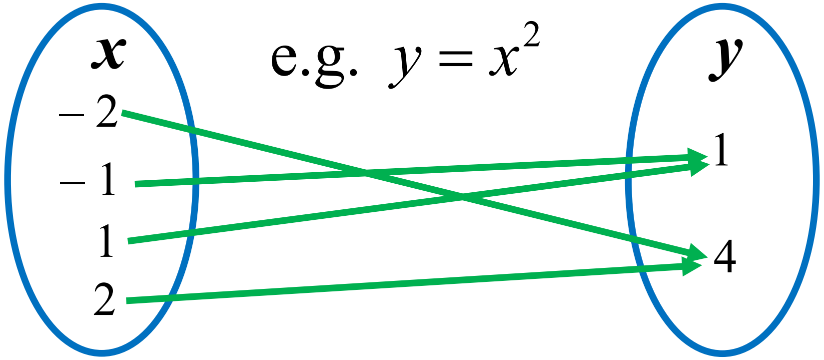
relation – a rule that maps between two sets of values

e.g. $x^2 + y^2 = 5$



function – a type of relation that uniquely maps from one set of values to another set of values

e.g. $y = x^2$



independent variable – the "***input***" of the function

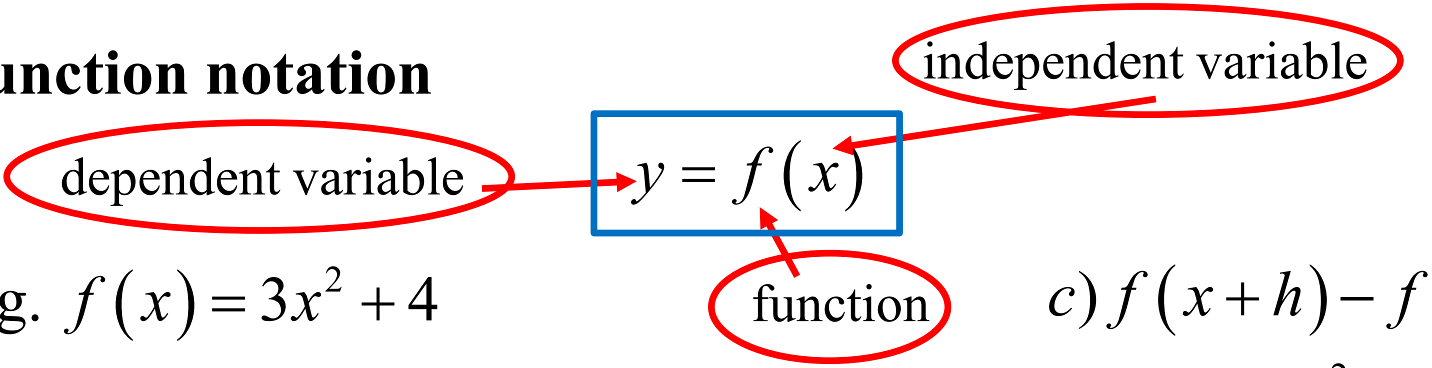
domain– *natural domain*: all possible values of the independent variable that can be substituted into the function

restricted domain: domains are restricted when some values are not required e.g. if independent variable represents time, then only $t \geq 0$ would be needed

dependent variable– the "***output***" of the function, its value *depends* upon the value of the independent variable

range– all possible values the dependent variable can take; obtained by substituting every value in the domain

function notation



independent variable

dependent variable

$$\boxed{y = f(x)}$$

function

e.g. $f(x) = 3x^2 + 4$

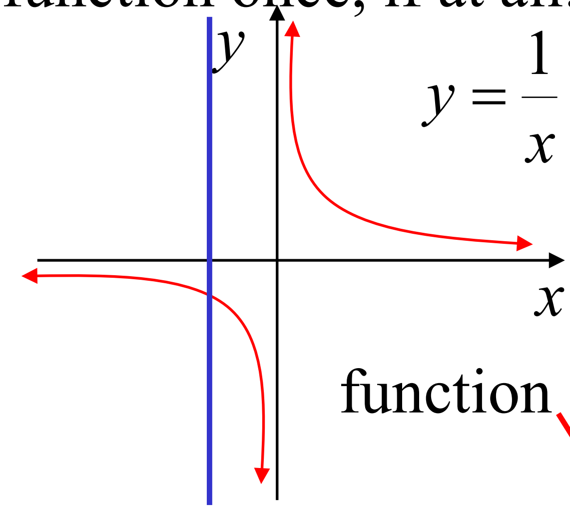
$$a)\ f(5) = 3(5)^2 + 4$$
$$= 75 + 4$$
$$\underline{= 79}$$

$$b)\ f(a) = \underline{3a^2 + 4}$$

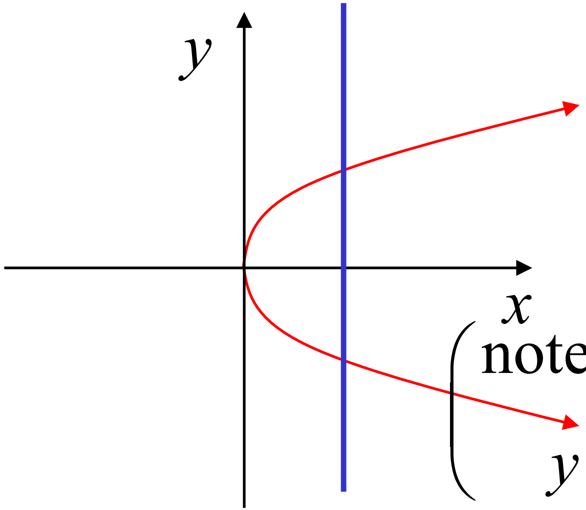
$$c)\ f(x+h) - f(x)$$
$$= 3(x+h)^2 + 4 - \left(3x^2 + 4\right)$$
$$= 3x^2 + 6xh + 3h^2 + 4 - 3x^2 - 4$$
$$\underline{= 6xh + 3h^2}$$

Vertical Line Test for Functions

If a straight line is drawn parallel to the y axis, it will only cross a function once, if at all.



function ✓



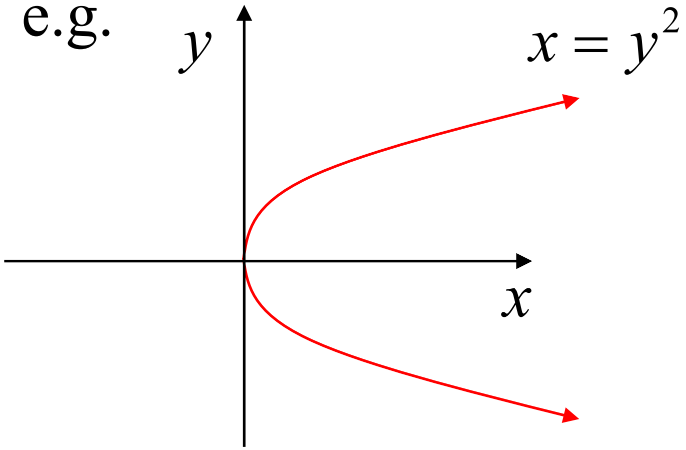
function ✗

(note: actually two functions $y = \sqrt{x}$ and $y = -\sqrt{x}$)

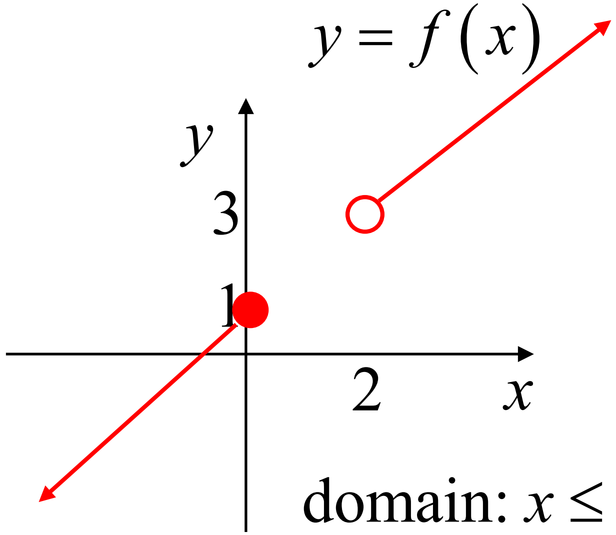
Domain and Range $y = f(x)$

Finding Domain: Geometrically it can be read from the graph

e.g.



domain: $x \geq 0$



domain: $x \leq 0$ and $x > 2$

Algebraically; things to look for:

1. Fractions: bottom of fraction $\neq$ 0

e.g. $(i)\ y = \frac{1}{x}$

$$x \neq 0$$

domain: all real x except $x = 0$

$(ii)\ y = \frac{1}{x^2 - 1}$

$$x^2 - 1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

domain: all real x except $x = \pm 1$

$(iii)\ y = \frac{4x}{x-1} + \frac{3}{7-x}$

$$x - 1 \neq 0 \qquad 7 - x \neq 0$$

$$x \neq 1 \qquad x \neq 7$$

domain: all real x except $x = 1$ or 7

2. Root Signs: you can't find the square root of a negative number.

e.g. $(i)\ y=\sqrt{4-x^2}$

$$4-x^2\geq 0$$

$$x^2\leq 4$$

domain: $-2\leq x\leq 2$

$(ii)\ y=\sqrt{x+3}-\sqrt{5-x}$

$x+3\geq 0$ $\quad$ $5-x\geq 0$

$x\geq -3$ $\quad$ $x\leq 5$

domain: $-3\leq x\leq 5$

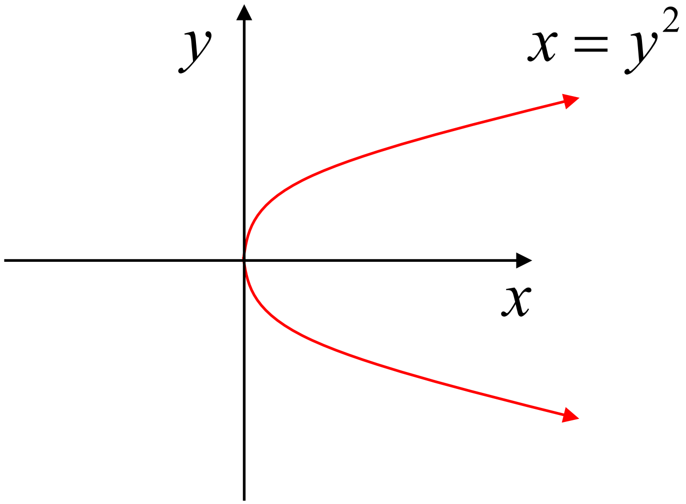
$(iii)\ y=\dfrac{1}{\sqrt{x+2}}$

$$x+2>0$$

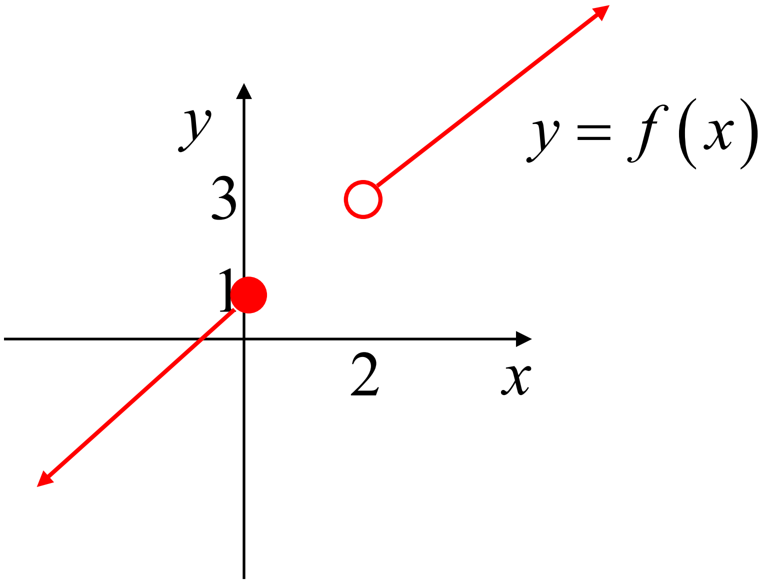
domain: $x>-2$

Range: Geometrically

e.g.



range: all real y



range: $y \le 1$ and $y > 3$

Algebraically; things to look for:

1. Maximum/Minimum values: even powers and absolute values are always $\geq$ 0

e.g. $(i)\ y = x^2$

range: $y \geq 0$

$(ii)\ y = x^2 + 3$

$y \geq 0 + 3$

range: $y \geq 3$

$(iii)\ y = 5 - x^2$

$y \leq 5 - 0$

range: $y \leq 5$

$(iv)\ y = |x + 2|$

range: $y \geq 0$

$(v)\ y = |x + 2| - 5$

$y \geq 0 - 5$

range: $y \geq -5$

2. Restrictions on Domain: sub in endpoints and centre of domain

e.g. $y=\sqrt{4-x^2}$ when $x=2, y=\sqrt{4-2^2}$ when $x=0, y=\sqrt{4-0^2}$

domain: $-2\leq x\leq 2$ $=0$ $=2$

range: $0\leq y\leq 2$

3. Fractions: If you have a constant on the top of the fraction, fraction $\neq 0$

e.g. (i) $y=\frac{1}{x}$

$y\neq 0$

range: all real y except $y=0$

(ii) $y=5+\frac{1}{x}$

$y\neq 5+0$

range: all real y except $y=5$

(iii) $y=\frac{x+7}{x+4}$

$$x+4\overset{\displaystyle 1}{\overline{\big)\,x+7}}$$

$$\underline{x+4}$$

$$3$$

$y=1+\frac{3}{x+4}$

$y\neq 1+0$

range: all real y except $y=1$

Exercise 3A; 1b, 2c, 3d, 4d, 6ac, 8b, 9b, 10, 11bd, 12bc, 13bd, 14ac, 15ac, 16, 17

Exercise 3B; 2, 3, 4b, 5d, 6b, 8be, 9, 10, 11, 13adf, 14, 15, 16