

The Quadratic Polynomial and the Parabola

Quadratic polynomial – $ax^2 + bx + c$

Quadratic function – $y = ax^2 + bx + c$

Quadratic equation – $ax^2 + bx + c = 0$

Coefficients – a, b, c

Indeterminate – x

Roots – Solutions to the quadratic equation

Zeroes – x intercepts of the quadratic function

e.g. Find the roots of $x^2 - 1 = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

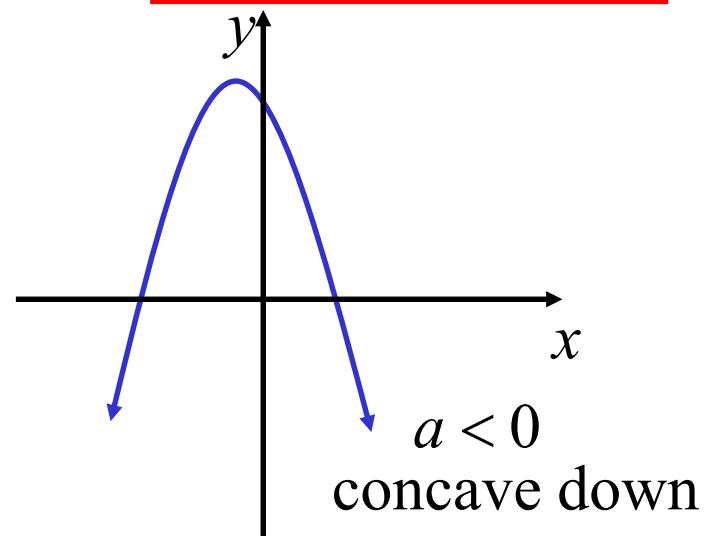
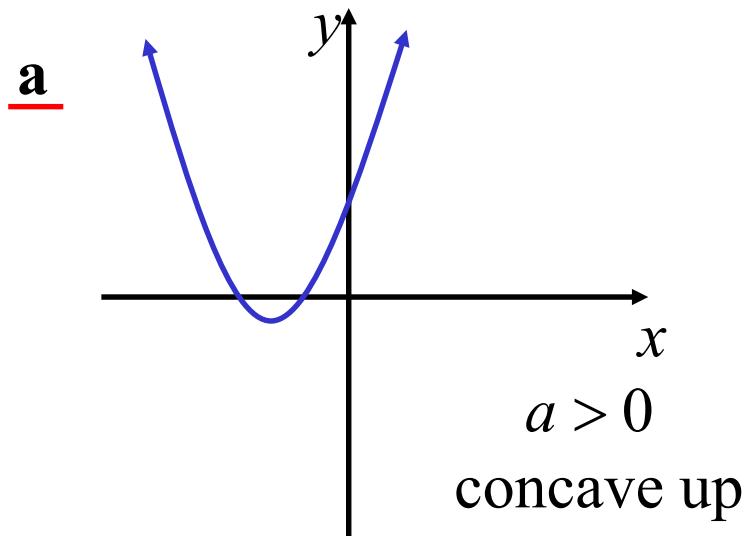
a quadratic with
 $a = 1$
is called **monic**

\therefore the roots are $x = -1$ and $x = 1$

Graphing Quadratics

The graph of a quadratic function is a parabola.

$$y = ax^2 + bx + c$$



c = y intercept

zeroes (roots) = x intercepts

$x = \frac{-b}{2a}$ = axis of symmetry

vertex x value is the AOS

y value is found by substituting AOS into the function.
(It is the maximum/minimum value of the function)

Note: AOS is the average of the zeroes

e.g. Graph $y = x^2 + 8x + 12$

a = 1 > 0 \therefore concave up

zeroes $x^2 + 8x + 12 = 0$

$$(x+6)(x+2) = 0$$

$$x = -6 \text{ or } x = -2$$

\therefore x intercepts are

$$(-6, 0) \text{ and } (-2, 0)$$

AOS $x = \frac{-b}{2a}$ *OR* $x = \frac{-6-2}{2}$

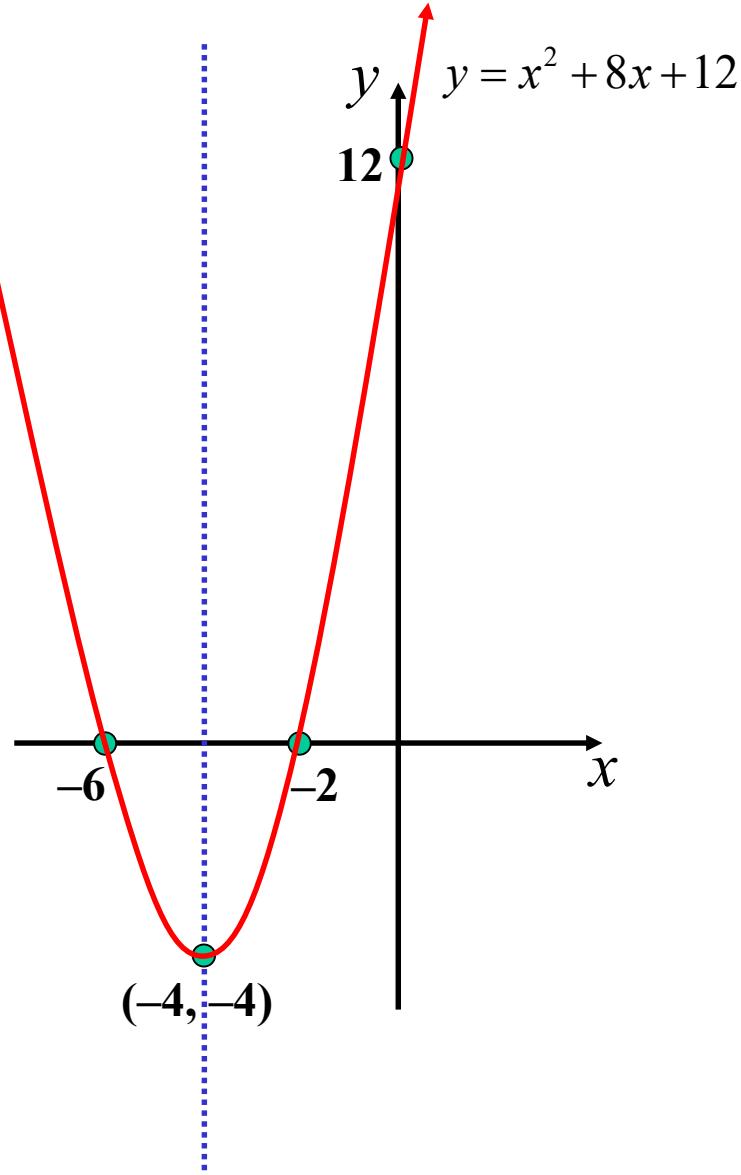
$$= \frac{-8}{2} = -4$$
$$= -4$$

vertex $y = (-4)^2 + 8(-4) + 12$

$$= -4$$

\therefore vertex is $(-4, -4)$

c = 12 \therefore y intercept is $(0, 12)$



(ii) Find the quadratic with;

a) roots 3 and 6

$$y = a(x^2 - 9x + 18)$$

$\xrightarrow{-(6+3)}$ $\xrightarrow{6 \times 3}$

b) monic roots $3 + \sqrt{2}$ and $3 - \sqrt{2}$

$$y = x^2 - 6x + 7$$

$\xrightarrow{-(3+\sqrt{2}+3-\sqrt{2})}$ $\xrightarrow{(3+\sqrt{2})(3-\sqrt{2})}$

c) roots 2 and 8 and vertex (5,3)

$$y = a(x^2 - 10x + 16)$$

$$(5,3): 3 = a(5^2 - 10(5) + 16)$$

$$3 = -9a$$

$$a = -\frac{1}{3}$$

$$\therefore y = -\frac{1}{3}(x^2 - 10x + 16)$$

**Exercise 3D; 1a, 2b(ii),
3a, 4 (use desmos),
5bd, 7bf, 9cf, 10c, 11ad,
12, 14, 15**