Equations of the form asinx + bcosx = c

Auxiliary Angle Method
(i) Change into a sine function

eg (i) $3\cos\theta + 4\sin\theta = 2$ $\quad\quad 0 \le \theta \le 360°$

$$\sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$4\sin\theta + 3\cos\theta = 2$$

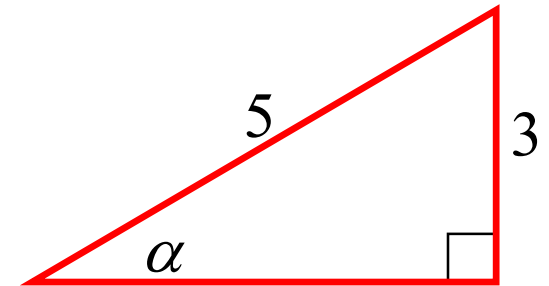$$5 \times \left(\frac{4}{5}\sin\theta + \frac{3}{5}\cos\theta \right) = 2$$

$$5\sin(\theta + \alpha) = 2$$

$$\sin(\theta + \alpha) = \frac{2}{5}$$

Q1, Q2

$$\sin\beta = \frac{2}{5}$$

$$\beta = 23°35'$$

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36°52'$$

$$\theta + 36°52' = 23°35', \ 156°25'$$

$$\theta = -13°17', 119°33'$$

$$\therefore \theta = 119°33', 346°43'$$

(ii) Change into a cosine function

eg (i) $3\cos\theta + 4\sin\theta = 2$ $\qquad 0 \le \theta \le 360°$

$$\cos\theta\cos\alpha + \sin\theta\sin\alpha$$

$$3\cos\theta + 4\sin\theta = 2$$

$$5 \times \left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta \right) = 2$$

$$5\cos(\theta - \alpha) = 2$$

$$\cos(\theta - \alpha) = \frac{2}{5}$$

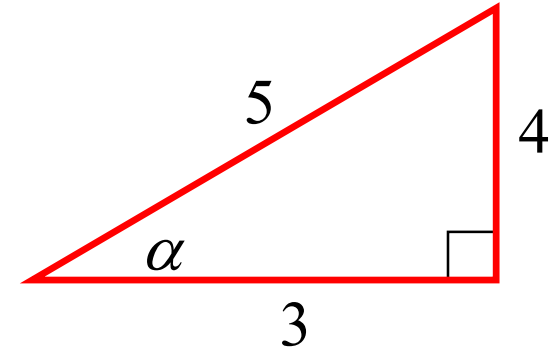Q1, Q4

$$\cos\beta = \frac{2}{5}$$

$$\beta = 66°25'$$

$$\theta - 53°8' = 66°25', 293°35'$$

$$\therefore \theta = 119°33', 346°43'$$

$$\tan\alpha = \frac{4}{3}$$

$$\alpha = 53°8'$$

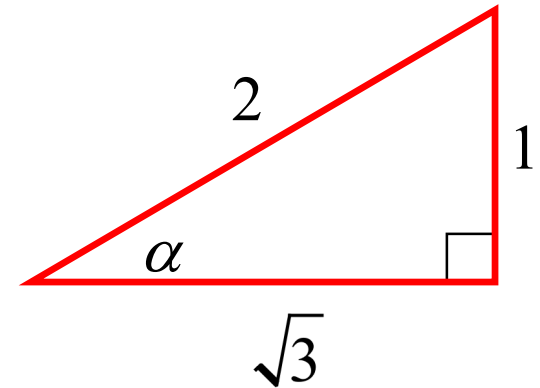(ii) $\sqrt{3}\sin x - \cos x = 1$ $0 \leq x \leq 2\pi$

$$2\sin\left(x - \frac{\pi}{6}\right) = 1$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

Q1, Q2

$$x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$$
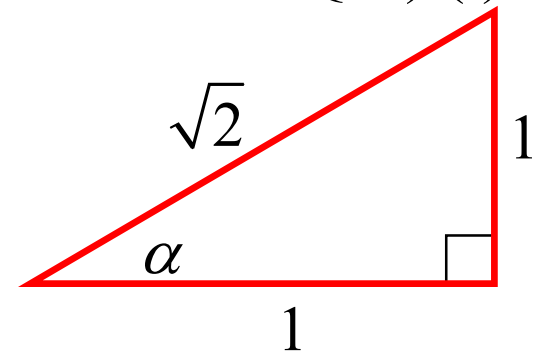
$$x = \frac{\pi}{3}, \pi$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6}$$

2003 Extension 1 HSC Q2e) (i)

(iii) Express $\cos x - \sin x$ in the form $R\cos(x + \alpha)$

$$\cos x - \sin x = \sqrt{2}\cos(x + 45°)$$

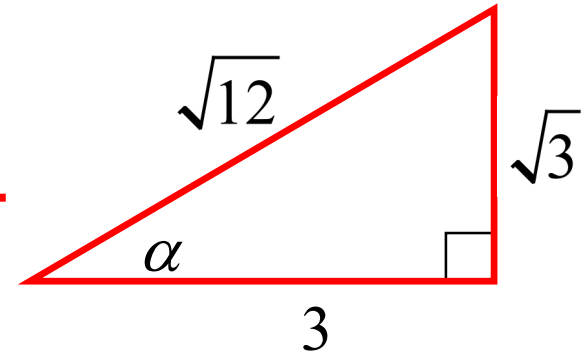
$$\tan\alpha = 1$$

$$\alpha = 45°$$

(iv) Express $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right)$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 \le \alpha \le \dfrac{\pi}{2}$

$$2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right) = 2\cos\theta + 2\cos\theta\cos\frac{\pi}{3} - 2\sin\theta\sin\frac{\pi}{3}$$

$$= 2\cos\theta + \cos\theta - \sqrt{3}\sin\theta$$

$$= 3\cos\theta - \sqrt{3}\sin\theta$$

$$= 2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right)$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6}$$

Exercise 11B;
6, 7, 10bd, 13ad, 14a, 15, 18, 19a, 20b