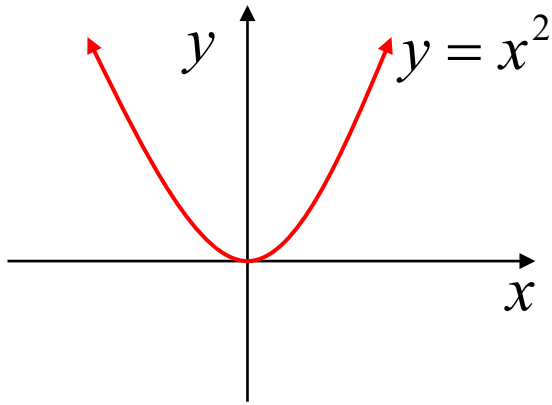


Quadratic Function



The linear function and the **quadratic function** are the building blocks of all polynomials

Every polynomial can be factorised down to a combination of linear and quadratic factors.

All quadratics can be transformed from the basic equation $y = x^2$ using translations, rotations, reflections or a combination of all three.

Recognising the quadratic function

$$y = ax^2 + bx + c$$

power '1' → (y) (ax²) → power '2'

- terms contain at most one variable, one variable is to the power of one, the other variable has a term to the power of two

Quadratics and Completing the Square

a measures concavity

$$y = a(x - h)^2 + k$$

vertex is (h, k)

x intercepts

$$(x + 4)^2 - 4 = 0$$

$$(x + 4)^2 = 4$$

$$x + 4 = \pm 2$$

$$x = -4 \pm 2$$

$$x = -6 \text{ or } x = -2$$

$\therefore x$ intercepts are

$$\underline{(-6, 0) \text{ and } (-2, 0)}$$

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$y = x^2 + 8x + 12$$

$$= (x + 4)^2 - 4$$

\therefore vertex is $(-4, -4)$

(ii) Write down the quadratic with roots 2 and 8 and vertex $(5, 3)$

$$y = k \left\{ (x - 5)^2 \right\} + 3$$

$$9k = -3$$

$$y = -\frac{1}{3} \left\{ (x - 5)^2 \right\} + 3$$

$$(2, 0): 0 = k \left\{ (2 - 5)^2 \right\} + 3$$

$$k = -\frac{1}{3}$$

$$\underline{y = -\frac{1}{3}(x^2 - 10x + 16)}$$

Quadratics and the Discriminant

$$\Delta = b^2 - 4ac$$

$$\text{vertex} = \left(\frac{-b}{2a}, \frac{-\Delta}{4a} \right)$$

$$\text{zeroes} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Note: if $\Delta < 0$, no x intercepts

$\Delta = 0$, one x intercept

$\Delta > 0$, two x intercepts

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$\Delta = 8^2 - 4(1)(12)$$

$$= 16$$

$$\therefore \text{vertex} = \left(-\frac{8}{2}, -\frac{16}{4} \right)$$

$$= \underline{(-4, -4)}$$

The Discriminant

The discriminant tells us whether the roots are rational or irrational

$\Delta > 0$: two different real roots (cuts the x axis twice)

$\Delta = 0$: two equal real roots (touches the x axis once)

$\Delta < 0$: no real roots (never touches the x axis)

Δ is a perfect square : roots are rational

e.g. (i) describe the roots of

$$2x^2 + 6x - 3 = 0$$

$$\Delta = 6^2 - 4(2)(-3)$$

$$= 60 > 0$$

\therefore two different, real, irrational roots

(ii) Find the values of k if

$$x^2 - 4x + 2k = 0 \text{ has no real roots}$$

unreal roots occur when $\Delta < 0$

$$\text{i.e. } (-4)^2 - 4(2k) < 0$$

$$16 - 8k < 0$$

$$\underline{k > 2}$$

**Exercise 3E; 1a, 2c, 3ace, 4b, 5ac, 6bc, 7c, 8, 9be,
10a, 11ace, 12be, 13ac, 14bdf, 15bc, 16, 19**

Exercise 3F; 1a, 2adf, 5a, 6, 7, 8, 9ace, 10a, 11, 12b, 14