

Translating Curves

Horizontal Shift replace x with $x - h$

$$\underline{y = f(x - h)}$$

moves $f(x)$ $\overset{-}{\leftarrow}$ or $\overset{+}{\rightarrow}$ h units

to find where the origin has been moved to; solve $x - h = 0$

Vertical Shift replace y with $y - k$

$$(y - k) = f(x) \quad \mathbf{OR} \quad \underline{y = f(x) + k}$$

moves $f(x)$ $\overset{+}{\uparrow}$ or $\overset{-}{\downarrow}$ k units

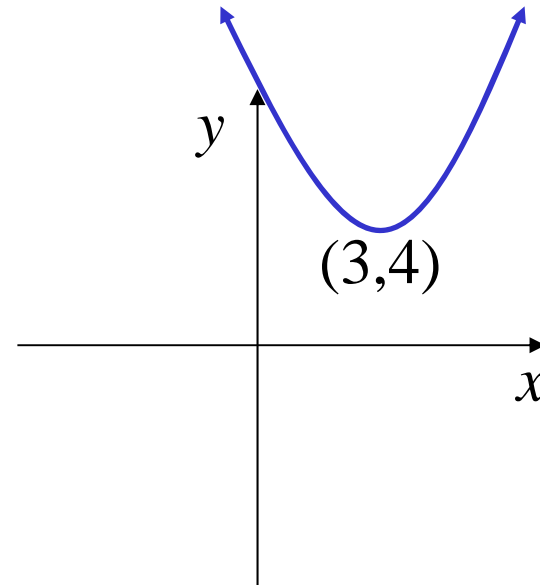
to find where the origin has been moved to; solve $y - k = 0$

e.g. (i) $y = (x - 3)^2 + 4$

1. *basic curve*: $y = x^2$

2. *shift right 3 units*

3. *shift up 4 units*



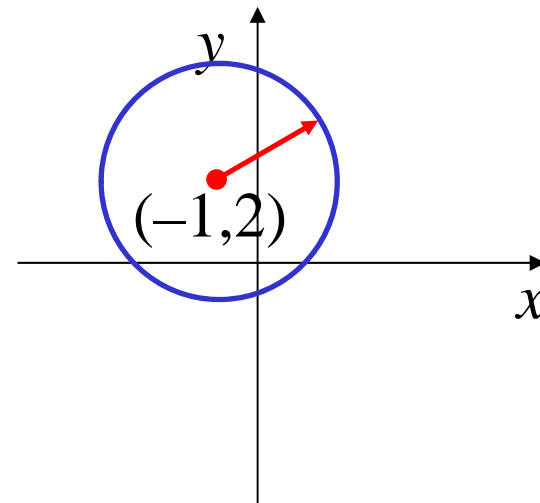
(ii) $x^2 + 2x + y^2 - 4y - 4 = 0$

$$\begin{aligned}(x + 1)^2 + (y - 2)^2 &= 4 + 1 + 4 \\ &= 9\end{aligned}$$

1. *basic curve*: $x^2 + y^2 = 9$

2. *shift left 1 unit*

3. *shift up 2 units*



$$(iii) y = 2x^2 + 8x + 5$$

$$\frac{y}{2} = x^2 + 4x + \frac{5}{2}$$

$$\frac{y}{2} = (x + 2)^2 + \frac{5}{2} - 4$$

$$= (x + 2)^2 - \frac{3}{2}$$

$$y = 2(x + 2)^2 - 3$$

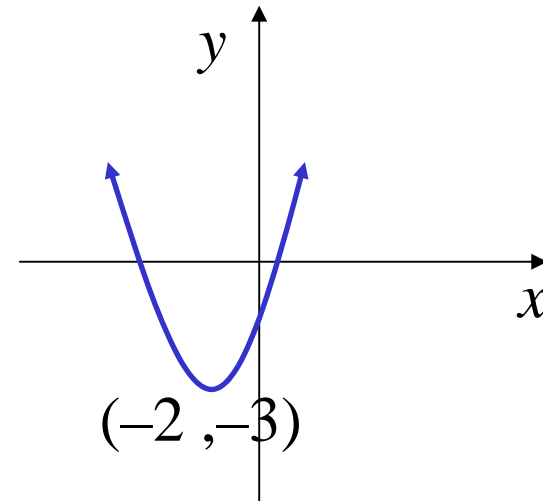
*It is easier to
complete the
square of x^2 than
 $2x^2$*

1. *basic curve: $y = x^2$*

2. *curve gets steeper*

3. *shift left 2 units*

4. *shift down 3 units*



Reflecting Curves

Vertical Reflection replace x with $-x$

$y = f(-x)$

Reflects in the y -axis

To reflect in the line $x = a$; replace x with $2a - x$

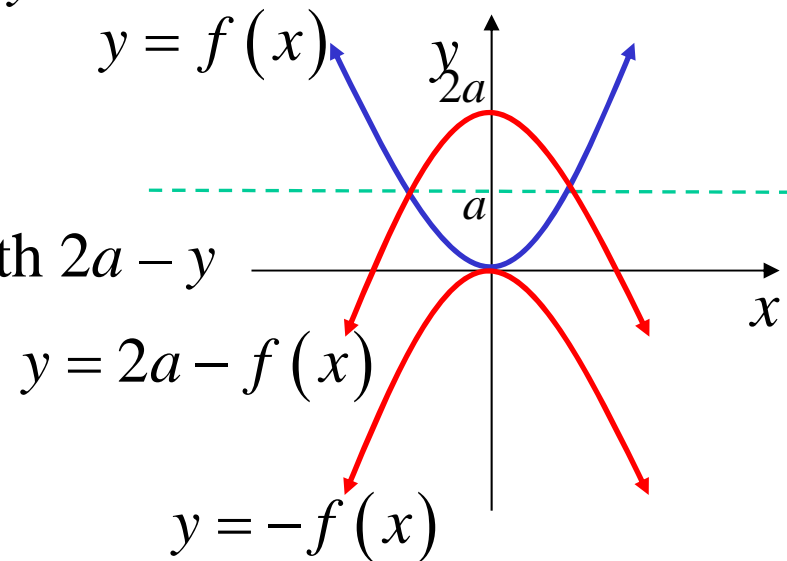
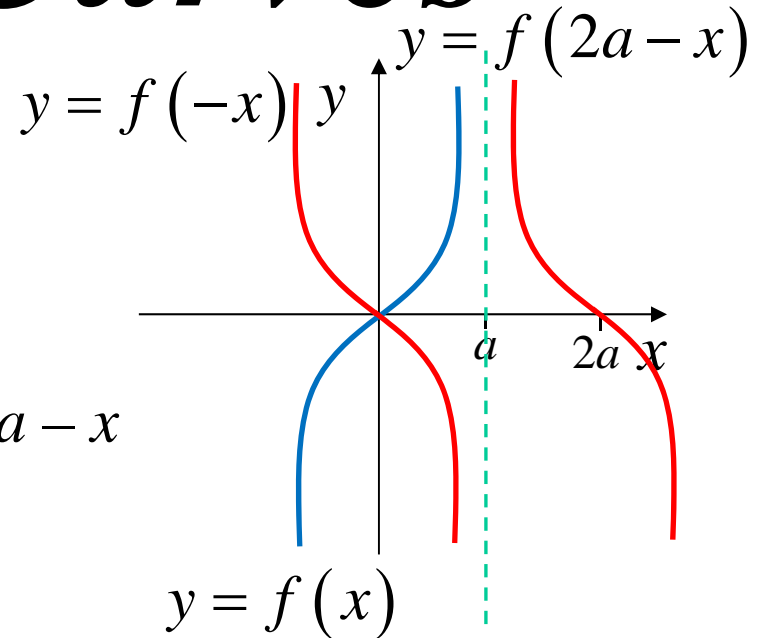
Horizontal Reflection replace y with $-y$

$-y = f(x)$ **OR** $y = -f(x)$

Reflects in the x -axis

To reflect in the line $y = a$; replace y with $2a - y$

i.e. $y = 2a - f(x)$

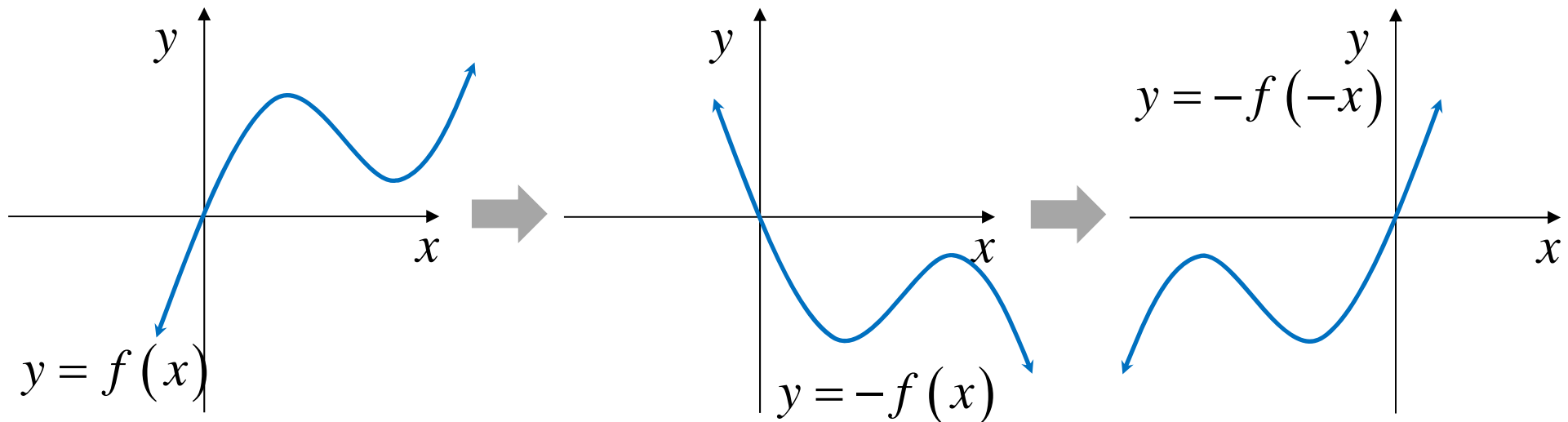


Reflection in the origin

$$\underline{y = -f(-x)}$$

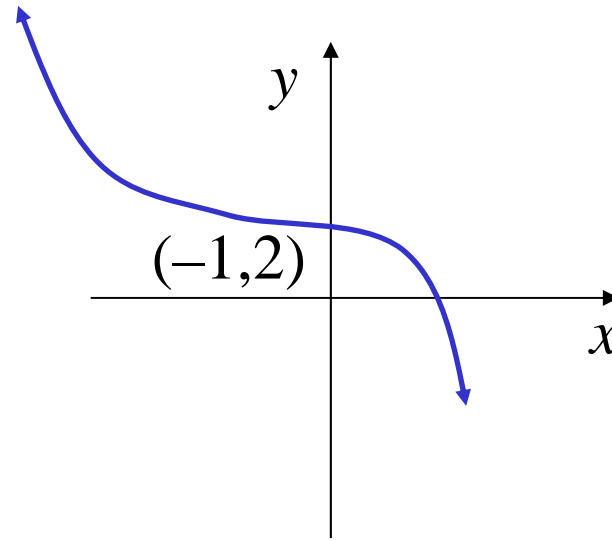
A reflection in the origin is the same as a rotation of 180°

It is achieved by two other reflections, one in the x -axis, the other in the y -axis



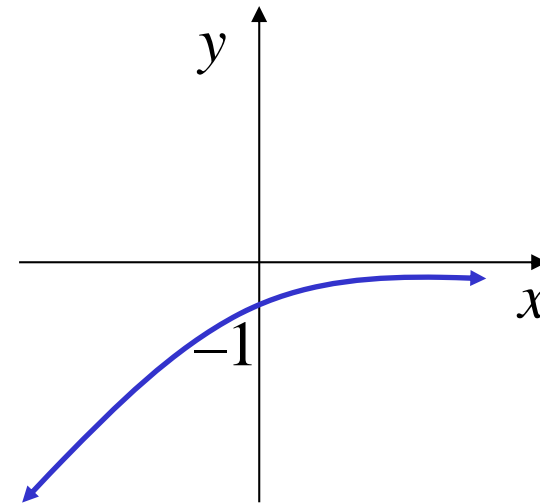
e.g. (i) $y = -(x+1)^3 + 2$

1. *basic curve*: $y = x^3$
2. *reflect in x axis*
3. *shift left 1 unit*
4. *shift up 2 units*



(ii) $y = -2^{-x}$

1. *basic curve*: $y = 2^x$
2. *reflect in origin*



**Exercise 4A; 3, 5bdefg, 6, 7abdf, 8acef,
9, 11ace, 12abdf, 13, 14c, 17, 18, 19**

Exercise 4B; 3, 5, 6d, 8, 10acdf, 11, 15