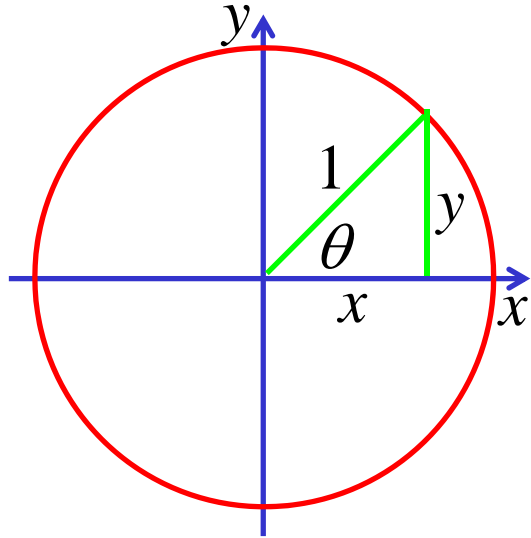


Pythagorean Trig Identities



$$x^2 + y^2 = 1$$

$$\sin \theta = \frac{y}{1}$$

$$\cos \theta = \frac{x}{1}$$

$$y = \sin \theta$$

$$x = \cos \theta$$

$$\therefore \underline{\sin^2 \theta + \cos^2 \theta = 1}$$

Divide by $\sin^2 \theta$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\underline{1 + \cot^2 \theta = \operatorname{cosec}^2 \theta}$$

Divide by $\cos^2 \theta$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\underline{\tan^2 \theta + 1 = \sec^2 \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

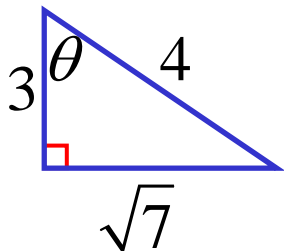
$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

In addition:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

e.g. (i) If $\cos \theta = \frac{3}{4}$ and $\tan \theta$ is negative, find $\sin \theta$



4th quadrant

$$\underline{\underline{\sin \theta = -\frac{\sqrt{7}}{4}}}$$

(ii) Simplify;

a) $1 - \cos^2 \theta$

$$= 1 - (1 - \sin^2 \theta)$$

$$= 1 - 1 + \sin^2 \theta$$

$$= \underline{\sin^2 \theta}$$

b) $\sec^2 A - \tan^2 A$

$$= 1 + \tan^2 A - \tan^2 A$$

$$= \underline{1}$$

c) $\tan \theta \cos \theta$

$$= \frac{\sin \theta}{\cos \theta} \times \frac{\cos \theta}{1}$$

$$= \underline{\sin \theta}$$

(iii) Prove $\frac{2\sec^2 \alpha - 2}{4 \tan \alpha} = \frac{\tan \alpha}{2}$

$$\frac{2\sec^2 \alpha - 2}{4 \tan \alpha} = \frac{2(1 + \tan^2 \alpha) - 2}{4 \tan \alpha}$$

$$= \frac{2 + 2 \tan^2 \alpha - 2}{4 \tan \alpha}$$

$$= \frac{2 \tan^2 \alpha}{4 \tan \alpha}$$

$$= \underline{\frac{\tan \alpha}{2}}$$

*when proving trig identities
you can;*

- 1. start with LHS and prove RHS*
- 2. start with RHS and prove LHS*
- 3. work on LHS and RHS
independently and show they
equal the same thing*

***NEVER SOLVE LIKE AN
EQUATION***

$$\begin{aligned}
 \text{OR } \frac{\tan \alpha}{2} &= \frac{\tan^2 \alpha}{2 \tan \alpha} \\
 &= \frac{\sec^2 \alpha - 1}{2 \tan \alpha} \\
 &= \frac{2 \sec^2 \alpha - 2}{4 \tan \alpha}
 \end{aligned}$$

$$\begin{aligned}
 \text{OR } \text{Prove } \frac{2 \sec^2 \alpha - 2}{4 \tan \alpha} - \frac{\tan \alpha}{2} &= 0 \\
 \frac{2 \sec^2 \alpha - 2}{4 \tan \alpha} - \frac{\tan \alpha}{2} &= \frac{2 \sec^2 \alpha - 2 - 2 \tan^2 \alpha}{4 \tan \alpha} \\
 &= \frac{2(1 + \tan^2 \alpha) - 2 - 2 \tan^2 \alpha}{4 \tan \alpha} \\
 &= \frac{2 + 2 \tan^2 \alpha - 2 - 2 \tan^2 \alpha}{4 \tan \alpha} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Thus } \frac{2 \sec^2 \alpha - 2}{4 \tan \alpha} - \frac{\tan \alpha}{2} &= 0 \\
 \therefore \frac{2 \sec^2 \alpha - 2}{4 \tan \alpha} &= \frac{\tan \alpha}{2}
 \end{aligned}$$

**Exercise 6F; 6abe, 7bdf, 8ab, 9bd,
10ad, 11, 13, 14**

**Exercise 6G; 3a, 4b, 5ad, 7ad,
8cd, 9c, 10ac, 11acegi, 12ace,
13bdfhj, 14bd, 15ac, 16acegh,
17a, 18a**