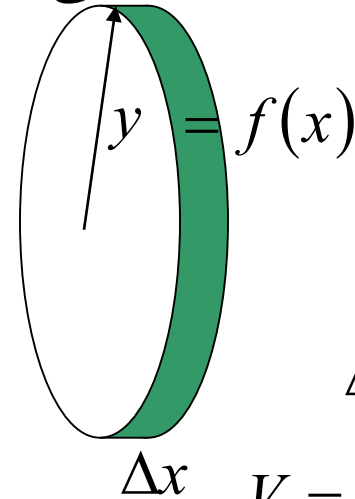
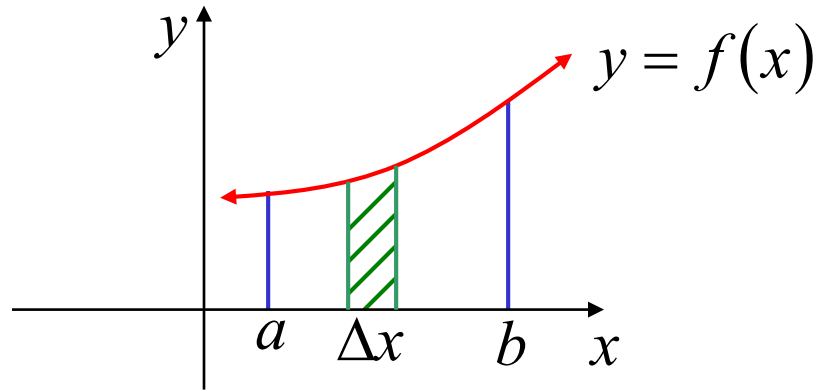


Volumes of Solids

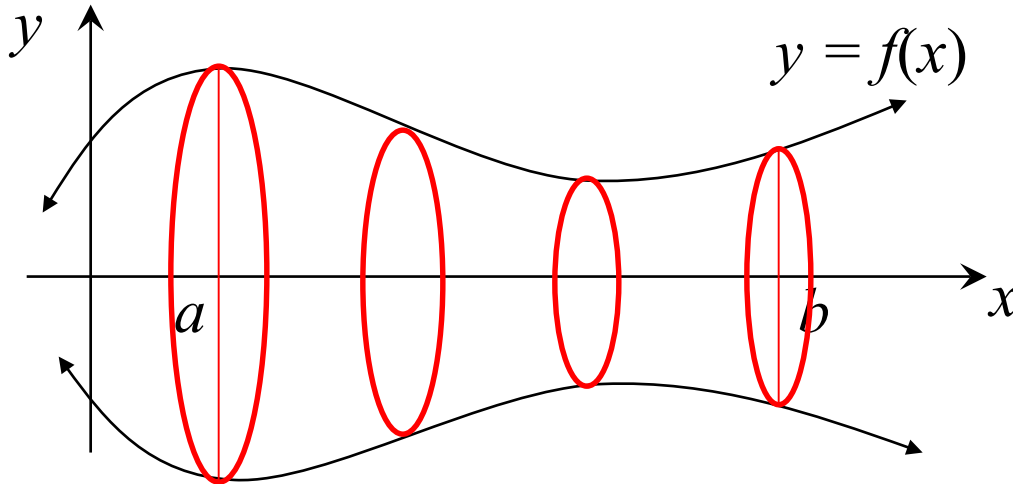


$$A(x) = \pi [f(x)]^2$$

$$\Delta V = \pi [f(x)]^2 \cdot \Delta x$$

$$V = \lim_{\Delta x \rightarrow 0} \sum_{x=a}^b \pi [f(x)]^2 \cdot \Delta x$$

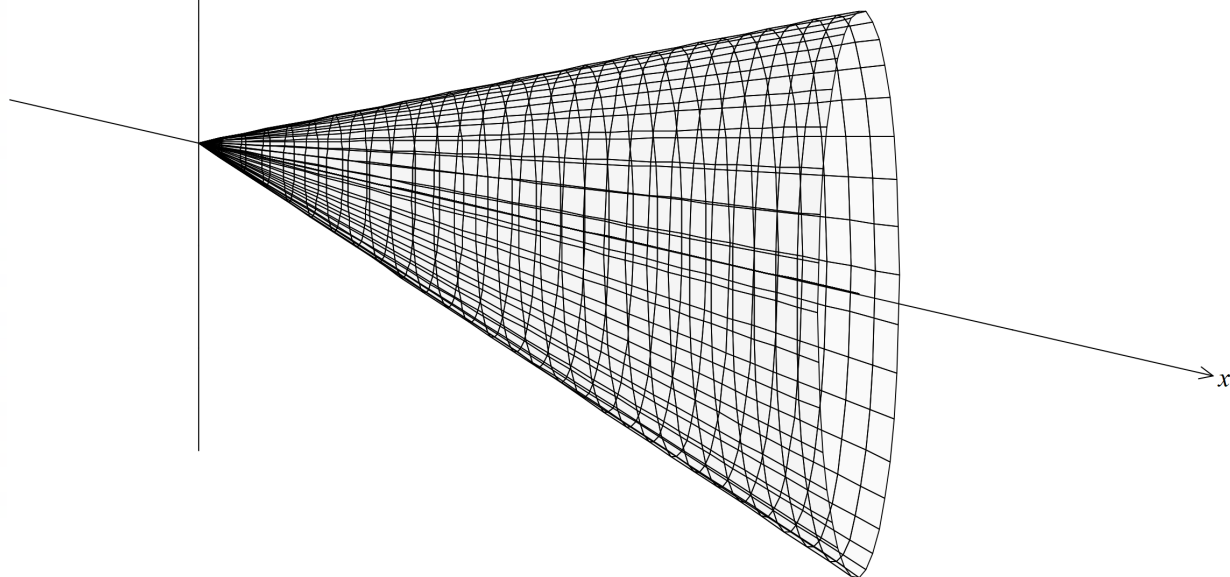
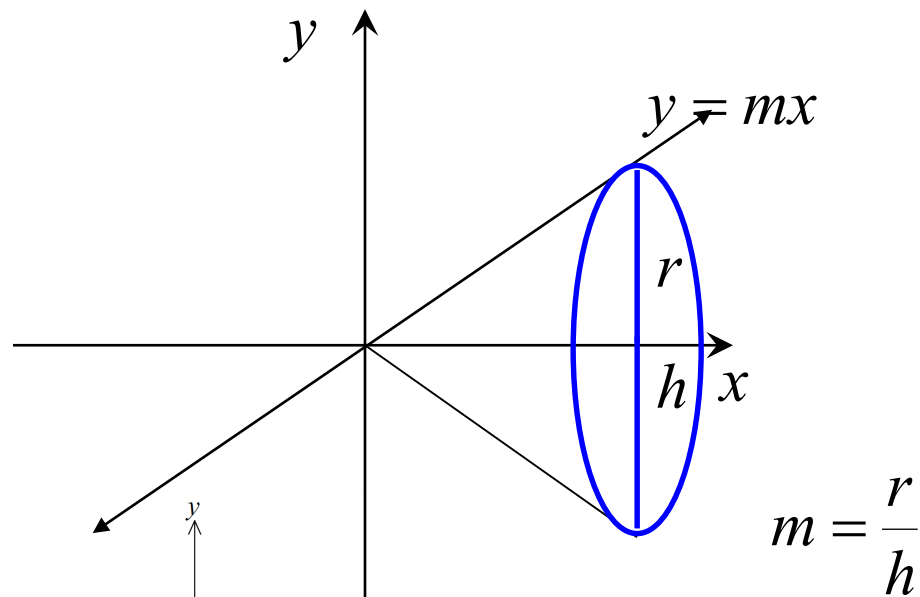
$$= \pi \int_a^b [f(x)]^2 dx$$



Volume of a solid of revolution about; (i) x axis : $V = \pi \int_a^b y^2 dx$

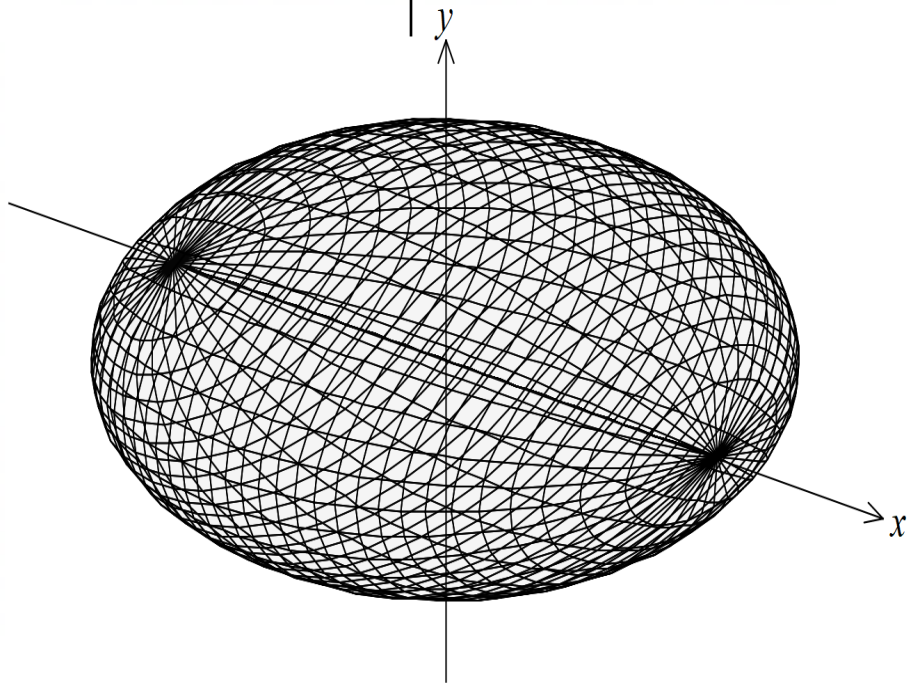
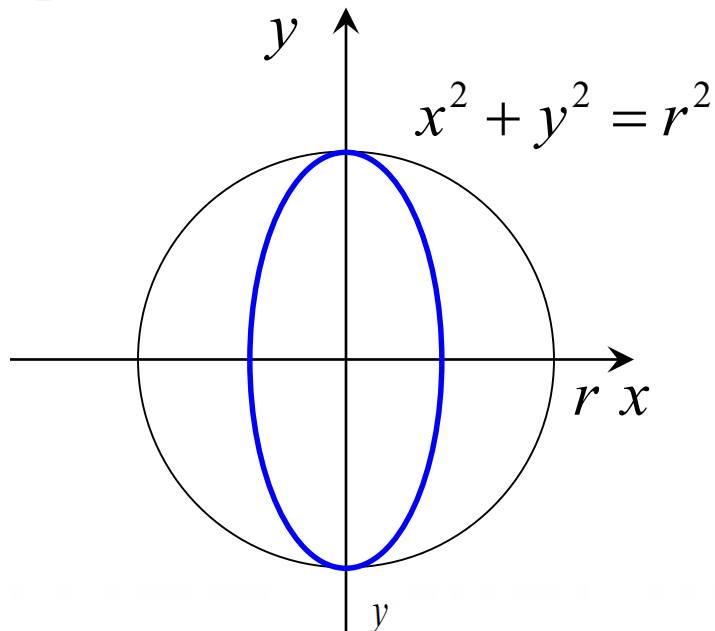
(ii) y axis : $V = \pi \int_c^d x^2 dy$

e.g. (i) cone



$$\begin{aligned} V &= \pi \int y^2 dx \\ &= \pi \int_0^h m^2 x^2 dx \\ &= \pi m^2 \left[\frac{1}{3} x^3 \right]_0^h \\ &= \frac{1}{3} \pi m^2 (h^3 - 0) \\ &= \frac{1}{3} \pi m^2 h^3 \\ &= \frac{1}{3} \pi \left(\frac{r}{h} \right)^2 h^3 \\ &= \frac{\pi r^2 h^3}{3h^2} \\ &= \frac{\pi r^2 h}{3} \end{aligned}$$

(ii) sphere



$$V = \pi \int y^2 dx$$

$$= 2\pi \int_0^r (r^2 - x^2) dx$$

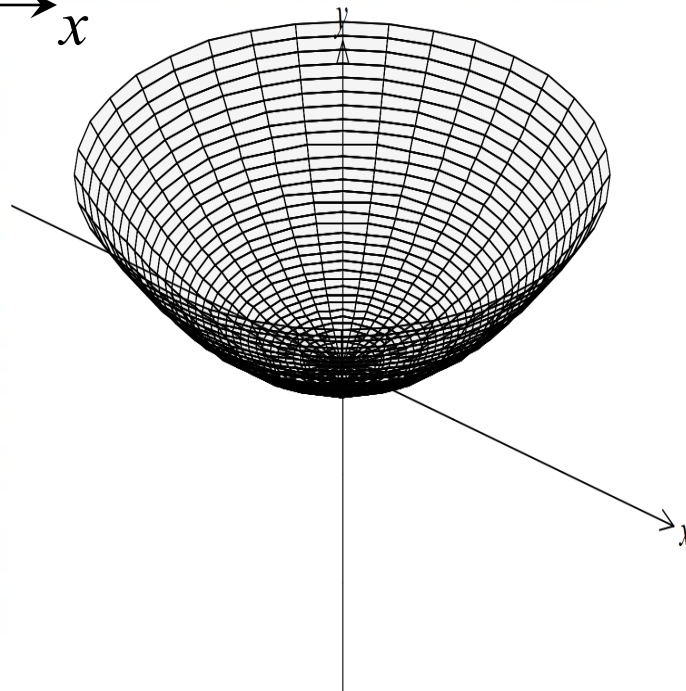
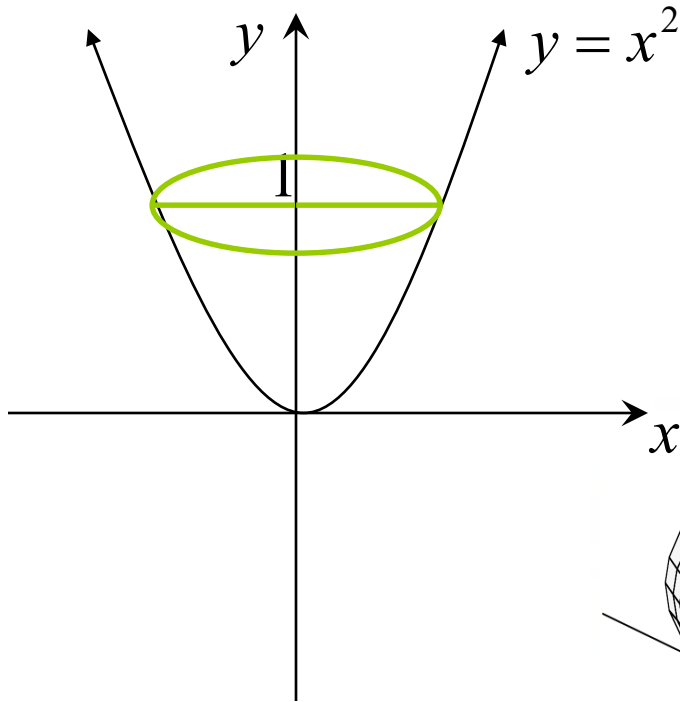
$$= 2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r$$

$$= 2\pi \left\{ \left(r^2(r) - \frac{1}{3} r^3 \right) - 0 \right\}$$

$$= 2\pi \left(\frac{2}{3} r^3 \right)$$

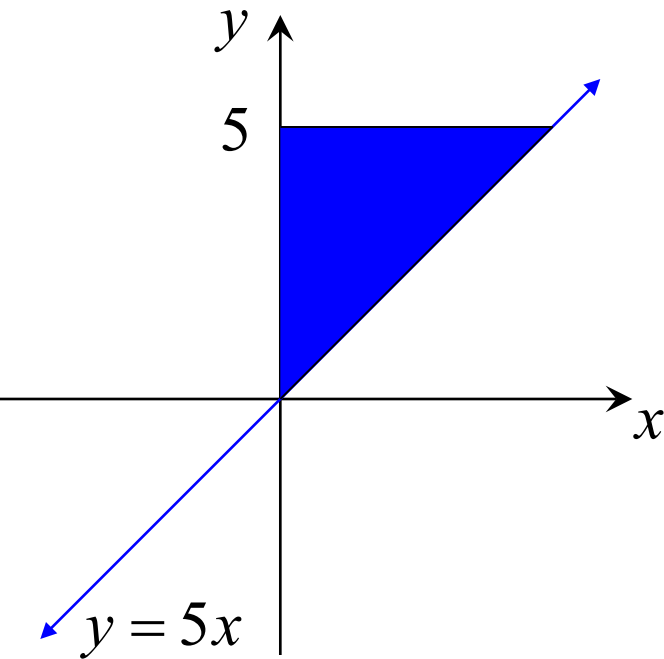
$$= \frac{4}{3} \pi r^3$$

(iii) Find the volume of the solid generated when $y = x^2$ is revolved around the y axis between $y = 0$ and $y = 1$.

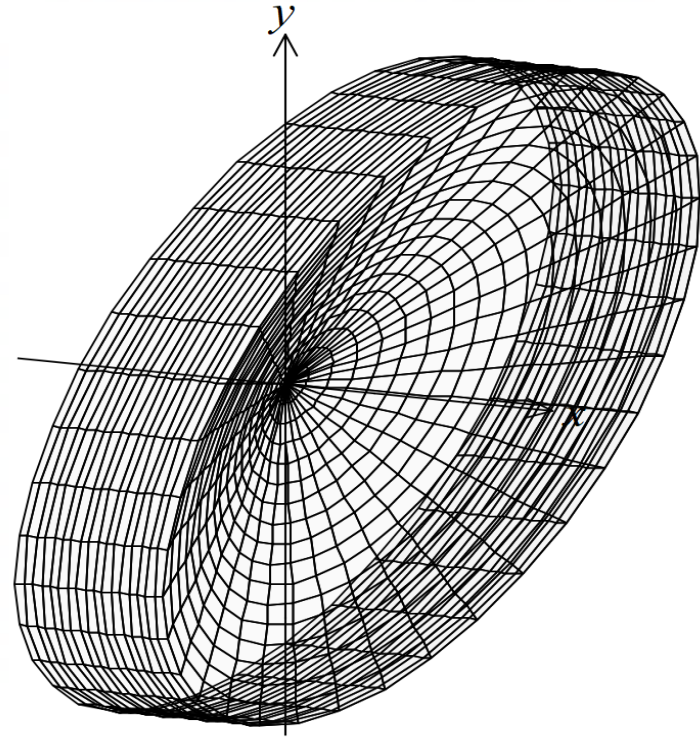


$$\begin{aligned} V &= \pi \int x^2 dy \\ &= \pi \int_0^1 y dy \\ &= \frac{\pi}{2} [y^2]_0^1 \\ &= \frac{\pi}{2} (1^2 - 0) \\ &= \frac{\pi}{2} \text{ units}^3 \end{aligned}$$

(iv) Find the volume of the solid when the shaded region is rotated about the x axis



$$\begin{aligned}
 V &= \pi \int y^2 dx \\
 &= \pi \int_0^1 \{5^2 - (5x)^2\} dx \\
 &= \pi \int_0^1 (25 - 25x^2) dx \\
 &= 25\pi \left[x - \frac{1}{3}x^3 \right]_0^1
 \end{aligned}$$

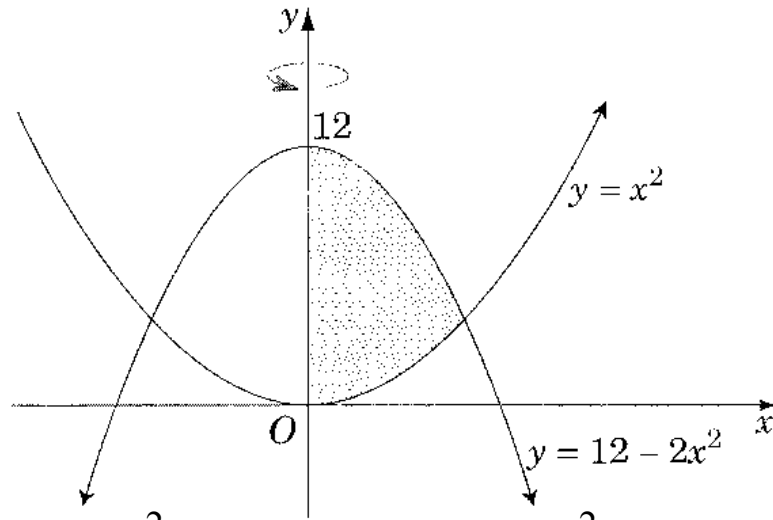


OR $V = \pi r^2 h - \frac{1}{3} \pi r^2 h$ $= 25\pi \left\{ 1 - \frac{1}{3}(1)^3 - 0 \right\}$

$$= \frac{2}{3} \pi (5)^2 (1) = \frac{50\pi}{3} \text{ unit}^3$$

$$= \frac{50\pi}{3} \text{ units}^3$$

2005 HSC Question 6c)



The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

(i) Find the points of intersection of the two curves.

(1)

$$x^2 = 12 - 2x^2$$

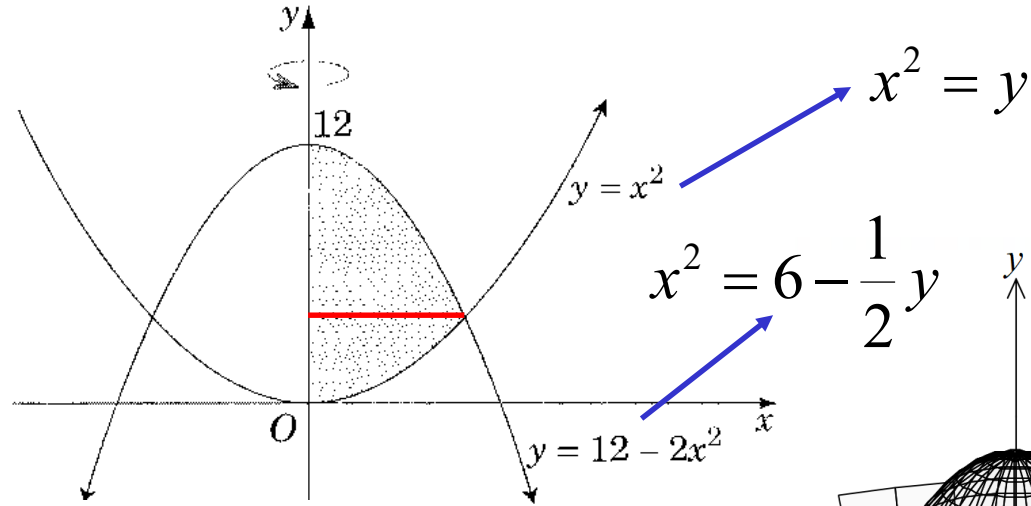
$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\therefore \text{meet at } \underline{(2, 4)}$$

(ii) The shaded region between the two curves and the y axis is rotated about the y axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. (3)



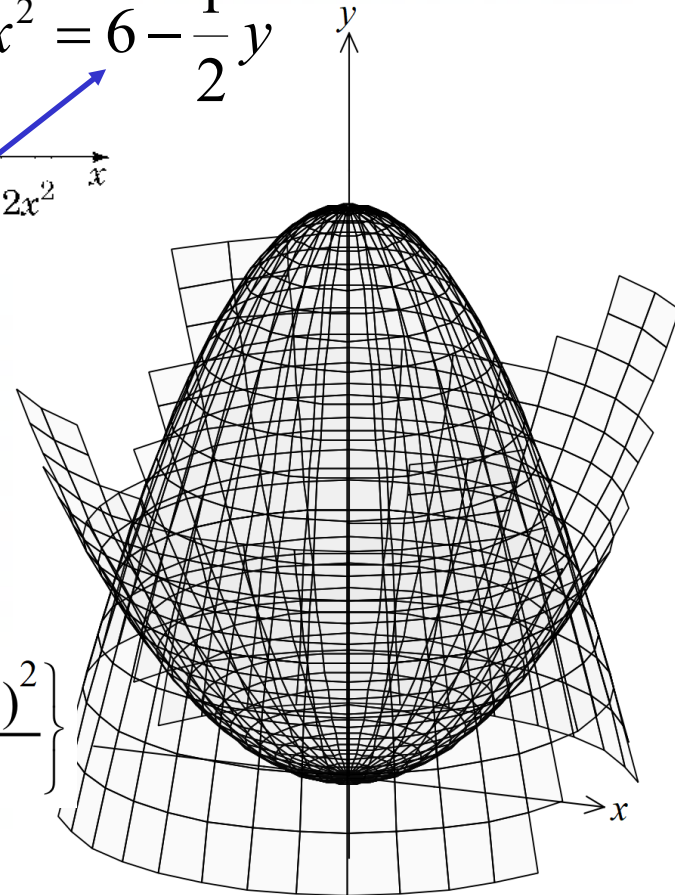
$$V = \pi \int x^2 dy$$

$$V = \pi \int_0^4 y dy + \pi \int_4^{12} \left(6 - \frac{1}{2} y \right) dy$$

$$= \pi \left[\frac{y^2}{2} \right]_0^4 - \pi \left[6y - \frac{y^2}{4} \right]_{12}^4$$

$$= \pi \left\{ \frac{(4)^2}{2} - 6(4) + \frac{(4)^2}{4} \right\} - \pi \frac{(0)^2}{2} + \pi \left\{ 6(12) - \frac{(12)^2}{4} \right\}$$

$$= \underline{24\pi \text{ units}^2}$$



**Exercise 11G; 3bdef, 4eg, 6, 9, 10cd, 11d, 13, 14,
16ad, 18, 20, 21, 23, 24, 25**