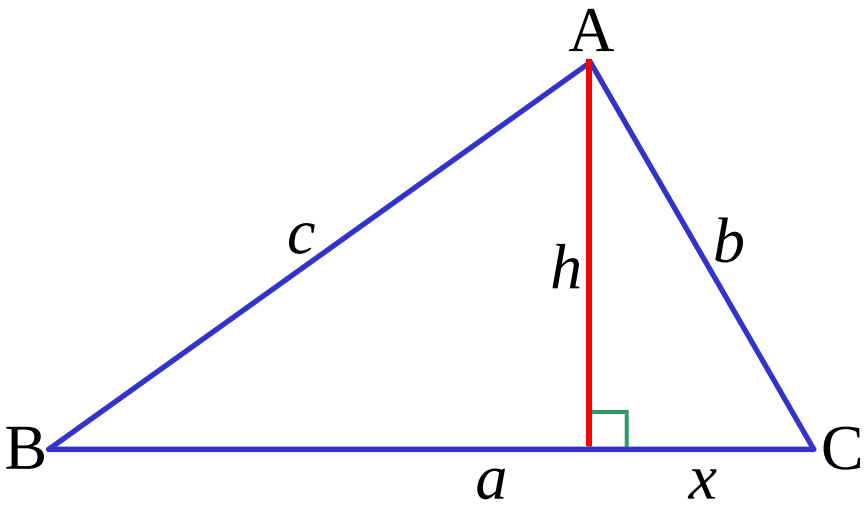
Cosine Rule



$$h^2 = b^2 - x^2$$

$$c^2 = h^2 + (a - x)^2$$

$$\therefore c^2 = b^2 - x^2 + a^2 - 2ax + x^2$$

$$= b^2 + a^2 - 2ax$$

$$\text{But } \frac{x}{b} = \cos C$$

$$x = b\cos C$$

$$\therefore c^2 = b^2 + a^2 - 2ab\cos C$$

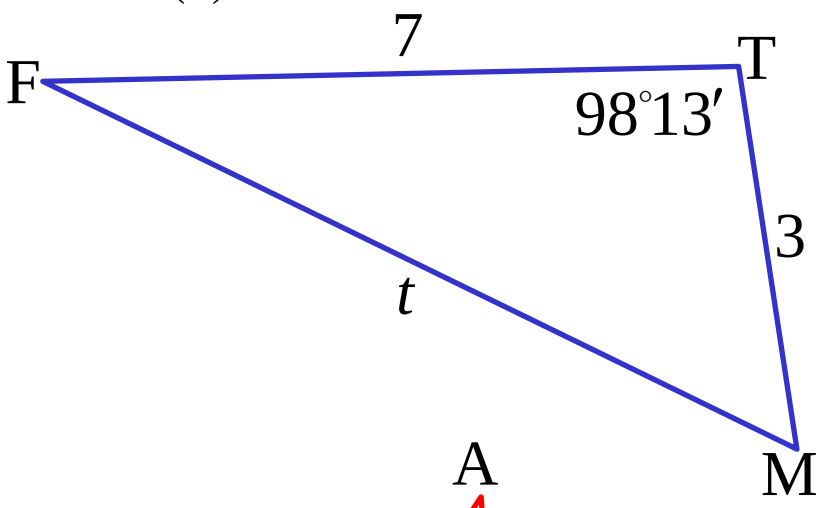
In any ΔABC

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

e.g. (i)

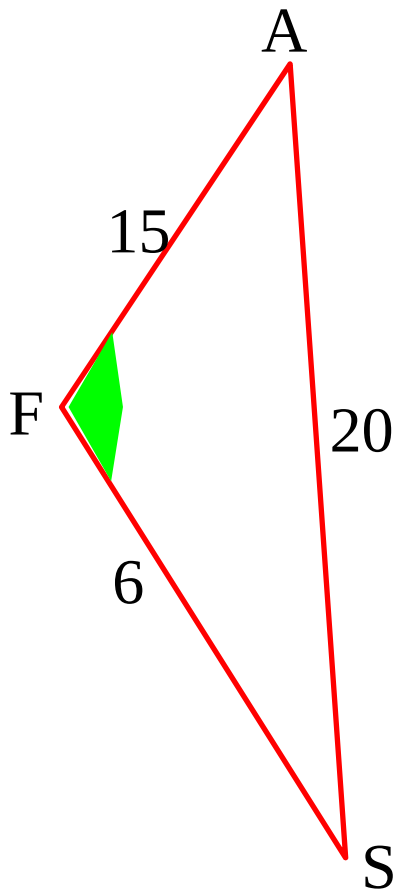


$$t^2 = f^2 + m^2 - 2fm\cos T$$

$$t^2 = 3^2 + 7^2 - 2(3)(7)\cos 98^\circ 13'$$

$t = 8$ units (to nearest whole number)

(ii)



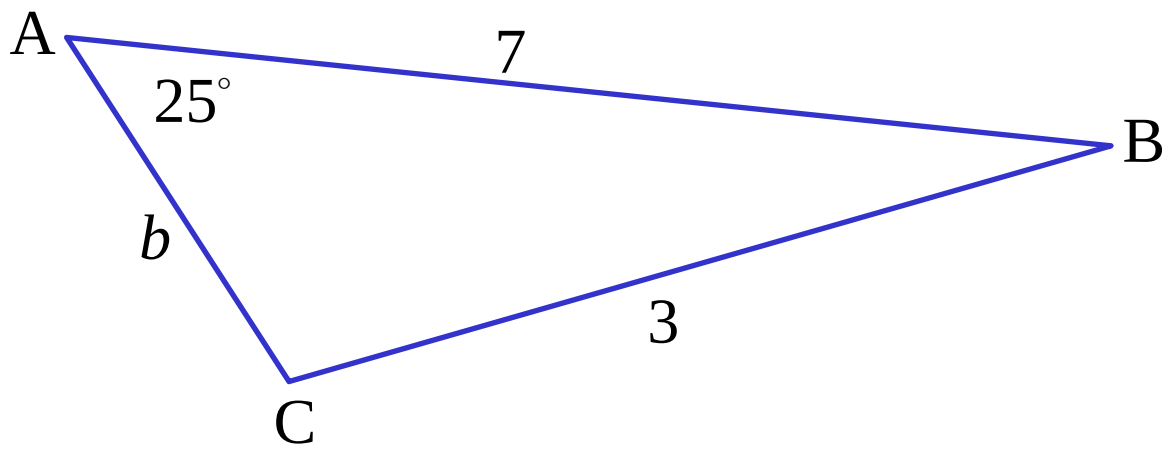
$$f^2 = a^2 + s^2 - 2as\cos F$$

$$\cos F = \frac{a^2 + s^2 - f^2}{2as}$$

$$\cos F = \frac{6^2 + 15^2 - 20^2}{2(6)(15)}$$

$F = 140^\circ 33'$

(iii)



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$3^2 = b^2 + 7^2 - 2b(7)\cos 25^\circ$$

$$9 = b^2 + 49 - 14b\cos 25^\circ$$

$$b^2 - 14b\cos 25^\circ + 40 = 0$$

$$b = \frac{14\cos 25^\circ \pm \sqrt{196\cos^2 25^\circ - 160}}{2}$$

$b = 5.85$ units or $b = 6.85$ units (to 2 dp)

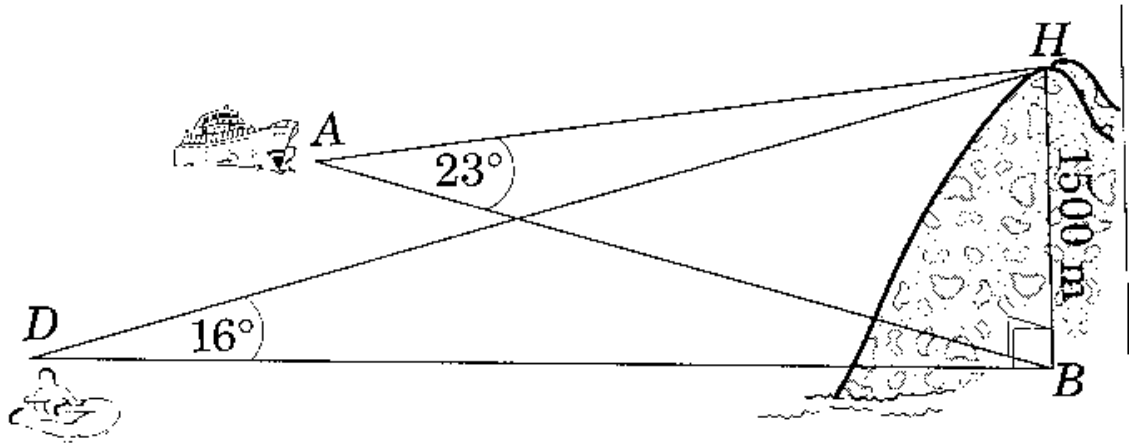
2003 Extension 1 HSC Q7a)

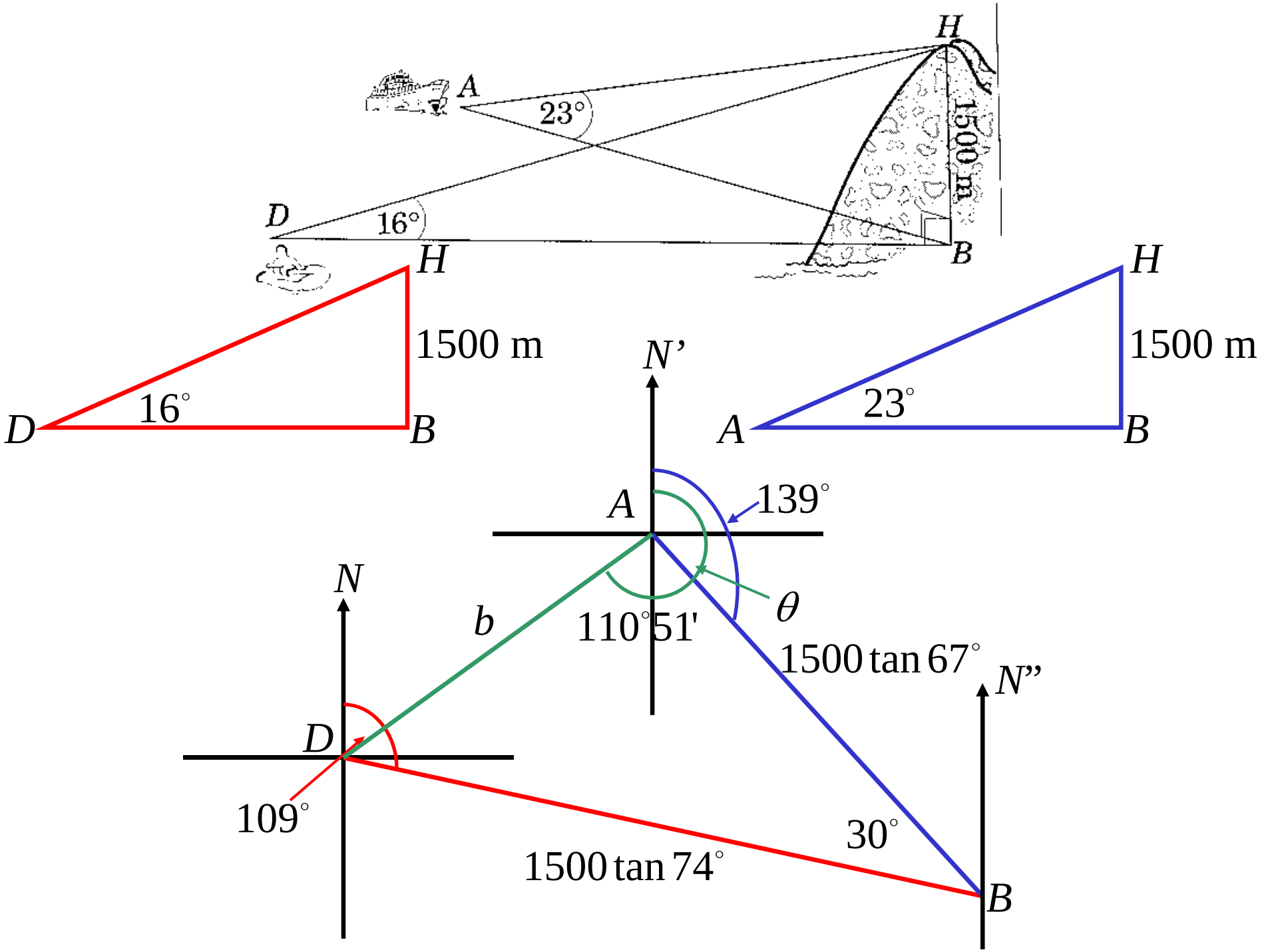
David is in a life raft and Anna is in a cabin cruiser searching for him. They are in contact by mobile phone. David tells Ana that he can see Mt Hope. From David's position the mountain has a bearing of 109°, and the angle of elevation to the top of the mountain is 16°.

Anna can also see Mt Hope. From her position it has a bearing of 139°, and and the top of the mountain has an angle of elevation of 23°.

The top of Mt Hope is 1500 m above sea level.

Find the distance and bearing of the life raft from Anna's position.




H
A
23°
1500 m
D
16°
H
B
1500 m
16°
D
B
N'
H
1500 m
23°
A
B
A
139°
N
θ
b
110°51'
1500 tan 67°
N"
D
109°
30°
1500 tan 74°
B

$$\frac{BD}{1500} = \tan 74^\circ$$

$$BD = 1500 \tan 74^\circ$$

Similarly;

$$AB = 1500 \tan 67^\circ$$

$$\angle NDB + \angle DBN'' = 180 \qquad (\text{cointerior } \angle\text{'s} = 180,\ \text{ND} \parallel \text{N''B})$$

$$109^\circ + \angle DBN'' = 180^\circ$$

$$\angle DBN'' = 71^\circ$$

Similarly;

$$\angle ABN'' = 41^\circ$$

$$\angle ABD = \angle DBN'' - \angle ABN'' \qquad (\text{common } \angle\text{'s})$$

$$\therefore \angle ABD = 30^\circ$$

$$b^2 = 1500^2 \tan^2 67^\circ + 1500^2 \tan^2 74^\circ - 2 \times 1500 \tan 67^\circ \times 1500 \tan 74^\circ \cos 30^\circ$$

$$b = 2798.96\ldots$$

$$= 2799 \text{ (to nearest metre)}$$

Anna and David are 2799 m apart.

Exercise 6J; 1e, 2f, 3a, 5b, 6, 7, 8, 11, 13, 15, 16, 17

Exercise 6K; 4, 5, 6, 10, 11, 12, 13, 14, 17, 19, 20

$$\frac{\sin \angle DAB}{1500 \tan 74^\circ} = \frac{\sin 30^\circ}{b}$$

$$\sin \angle DAB = \frac{1500 \tan 74^\circ \sin 30^\circ}{b}$$

$$\angle DAB = 69^\circ 9' \quad \text{or} \quad 110^\circ 51'$$

If $\angle DAB = 69^\circ 9'$

then $\angle BDA = 80^\circ 51'$

But $\angle DAB > \angle BDA$

$\therefore \angle BDA = 110^\circ 51'$

$\therefore$ The bearing of David from Anna is $249^\circ 51'$