Cartesian Geometry

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

e.g. Find the distance between (–1,3) and (3,5)

$$d = \sqrt{(5-3)^2 + (3+1)^2}$$

$$= \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

The distance formula is finding the length of the hypotenuse, using Pythagoras

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

e.g. Find the midpoint of (3,4) and (–2 ,1)

$$M = \left(\frac{3-2}{2}, \frac{4+1}{2} \right)$$

$$= \left(\frac{1}{2}, \frac{5}{2} \right)$$

The midpoint formula averages the x and y values

Using Euclidean Geometry with Cartesian Geometry

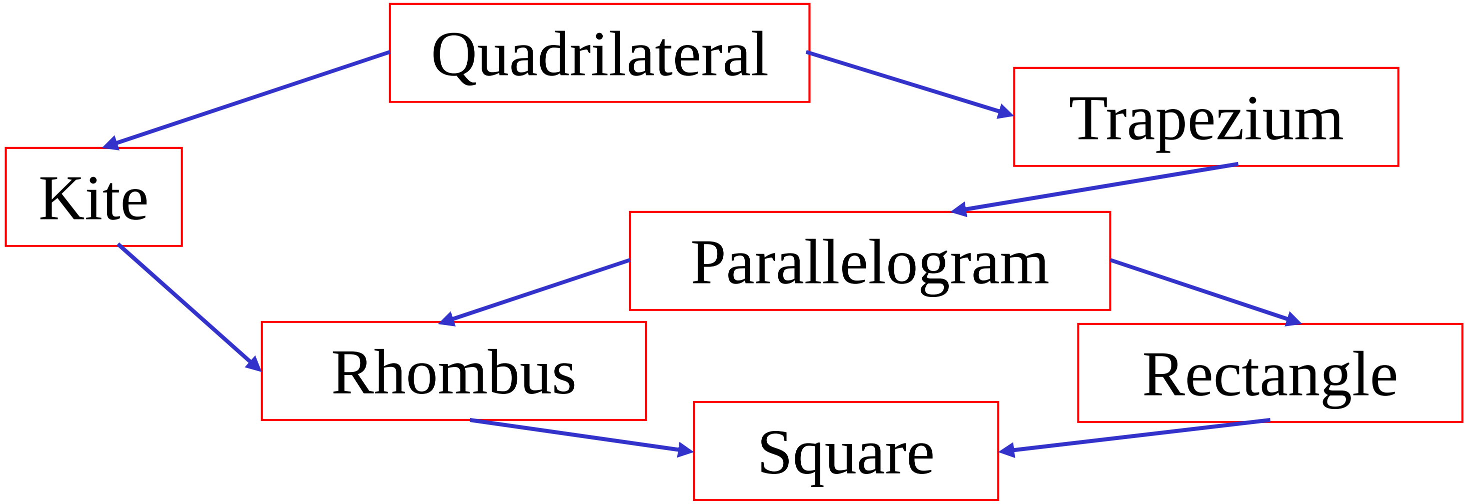
The ideas (theorems) from Euclidean Geometry are often used to solve problems in Cartesian Geometry.

Classifying Quadrilaterals

Quadrilaterals can be classified according to the features they possess. As you branch down the quadrilateral family, the quadrilaterals become more specialised in their features.

Any property that you know about the quadrilaterals a particular shape has branched from can be used for that quadrilateral as well as the specific properties for that shape.

e.g. a rhombus has the properties of a rhombus, parallelogram, trapezium, kite and quadrilateral.



Quadrilateral

* has 4 sides
* has 4 angles
* no specific properties

Kite

* adjacent sides in a kite are =
* diagonals in a kite are $\perp$
* Area $= \frac{1}{2}xy$

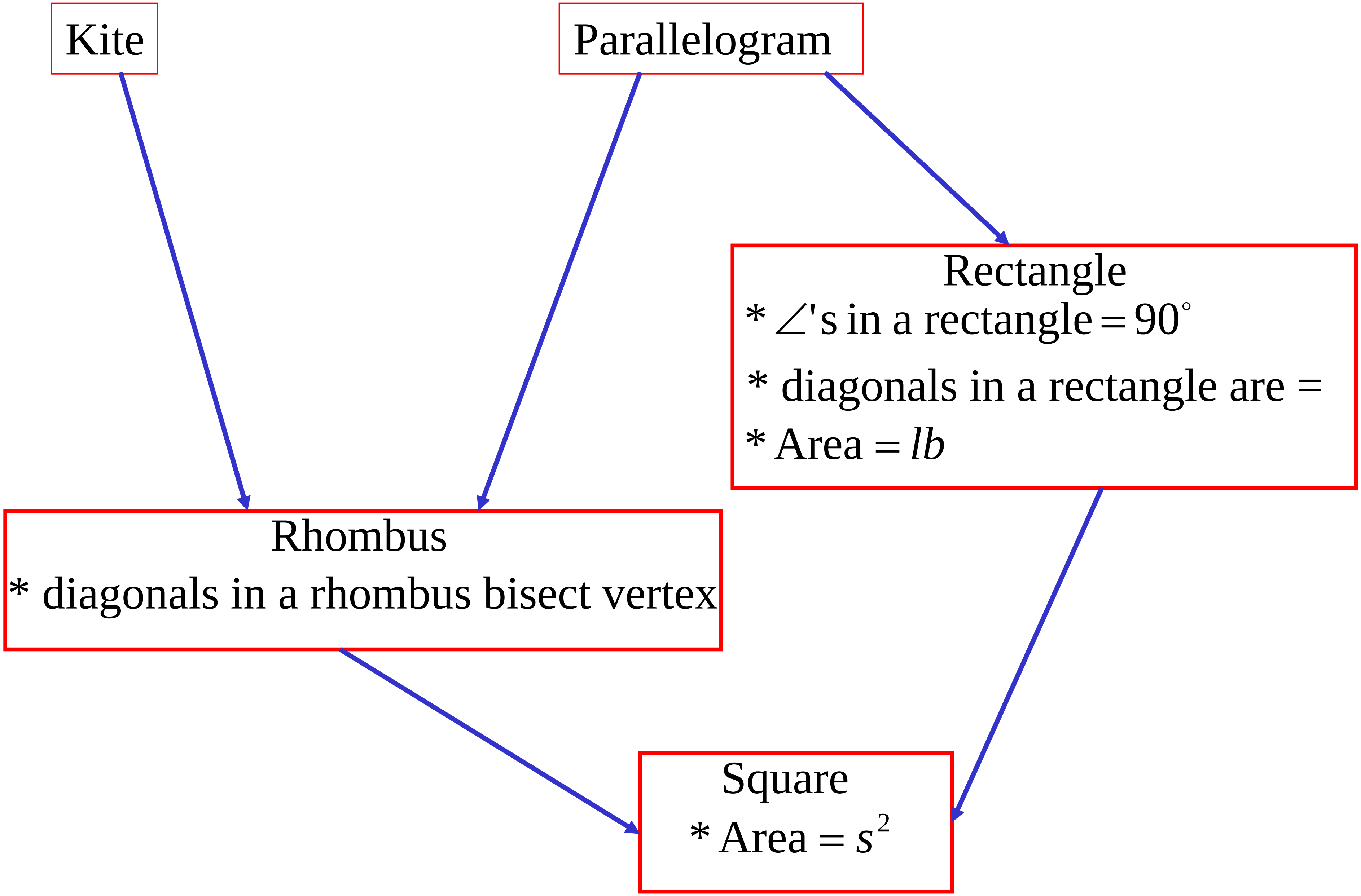
Trapezium

* one pair of parallel sides
* Area $= \frac{1}{2}h(a+b)$

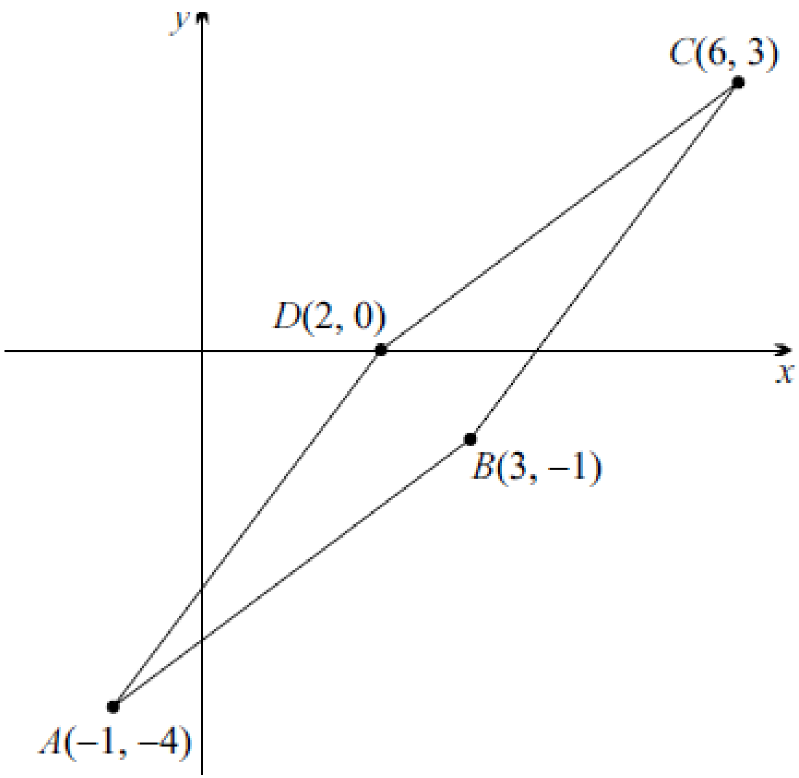
Parallelogram

* two pairs of parallel sides
* opposite sides in a || gram are =
* opposite $\angle$'s in a || gram are =
* diagonals in a || gram bisect
* Area $= bh$

NOTE: if one pair of sides are both = and parallel, then shape is ||gram


Kite
Parallelogram
Rectangle
* ∠'s in a rectangle = 90°
* diagonals in a rectangle are =
* Area = lb
Rhombus
* diagonals in a rhombus bisect vertex
Square
* Area = s²

e.g. The points $A(-1,-4), B(3,-1), C(6,3)$ and $D(2,0)$ form a quadrilateral as shown.



Show that $ABCD$ is a rhombus

$$M_{AC} = \left(\frac{6-1}{2}, \frac{3-4}{2}\right)$$

$$= \left(\frac{5}{2}, -\frac{1}{2}\right)$$

$$M_{BD} = \left(\frac{2+3}{2}, \frac{0-1}{2}\right)$$

$$= \left(\frac{5}{2}, -\frac{1}{2}\right) = M_{AC}$$

$\therefore ABCD$ is a parallelogram (diagonals bisect)

$$d_{CD} = \sqrt{(6-2)^2 + (3-0)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5$$

$$d_{AD} = \sqrt{(-1-2)^2 + (-4-0)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= 5 \quad = d_{CD}$$

$\therefore ABCD$ is a rhombus (parallelogram with adjacent sides =)

Exercise 7A; 1ad, 2be, 4, 5, 7bc, 8, 9, 10, 12b, 14d, 15, 16ac, 17, 18