Finding Limits

(1) Direct Substitution

e.g. $\lim\limits_{x \to 5} x + 7 = 5 + 7$

$$= \underline{12}$$

(2) Factorise and Cancel

e.g. $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3} = \lim\limits_{x \to 3} \dfrac{(x+3)(x-3)}{(x-3)}$

$$= \lim\limits_{x \to 3} (x + 3)$$

$$= 3 + 3$$

$$= \underline{6}$$

(3) Special Limit

$$\boxed{\lim_{x \to \infty} \frac{1}{x} = 0}$$

e.g. (i) $\displaystyle\lim_{x \to \infty} \frac{x^3 + 3x^2 + 2x - 1}{4x^3 - 1}$ $= \displaystyle\lim_{x \to \infty} \dfrac{\dfrac{x^3}{x^3} + \dfrac{3x^2}{x^3} + \dfrac{2x}{x^3} - \dfrac{1}{x^3}}{\dfrac{4x^3}{x^3} - \dfrac{1}{x^3}}$

$$= \lim_{x \to \infty} \frac{1 + \dfrac{3}{x} + \dfrac{2}{x^2} - \dfrac{1}{x^3}}{4 - \dfrac{1}{x^3}}$$

$$= \underline{\frac{1}{4}}$$

(ii) $\displaystyle\lim_{x \to \infty} \frac{4x - x^2}{x^3 + 1} = \frac{0}{1}$

$$= \underline{0}$$

(iii) $\displaystyle\lim_{x \to \infty} \frac{x^7 + x^6 + x^2}{3x^7 - x - 974} = \underline{\frac{1}{3}}$

(iv) $\displaystyle\lim_{x\to\infty}\frac{x^3+2}{x^2-1} = \frac{1}{0}$

$$= \infty$$

(v) Find the horizontal asymptote of $y = \dfrac{(x+3)(x-2)}{(x-1)(x+1)}$

$$\lim_{x\to\infty}\frac{(x+3)(x-2)}{(x-1)(x+1)} = \lim_{x\to\infty}\frac{x^2+x-6}{x^2-1}$$

$$= \frac{1}{1}$$

$$= 1$$

$\therefore$ horizontal asymptote is $y=1$

"Terry Lee"
Exercise 6.1