Slope Fields

The **slope field**, (direction field, gradient field), of a differential equation $\frac{dy}{dx} = f(x,y)$ assigns to each point $P(x,y)$ in the plane the number $f(x,y)$, which is the gradient of the solution curve through P.

creating a slope field

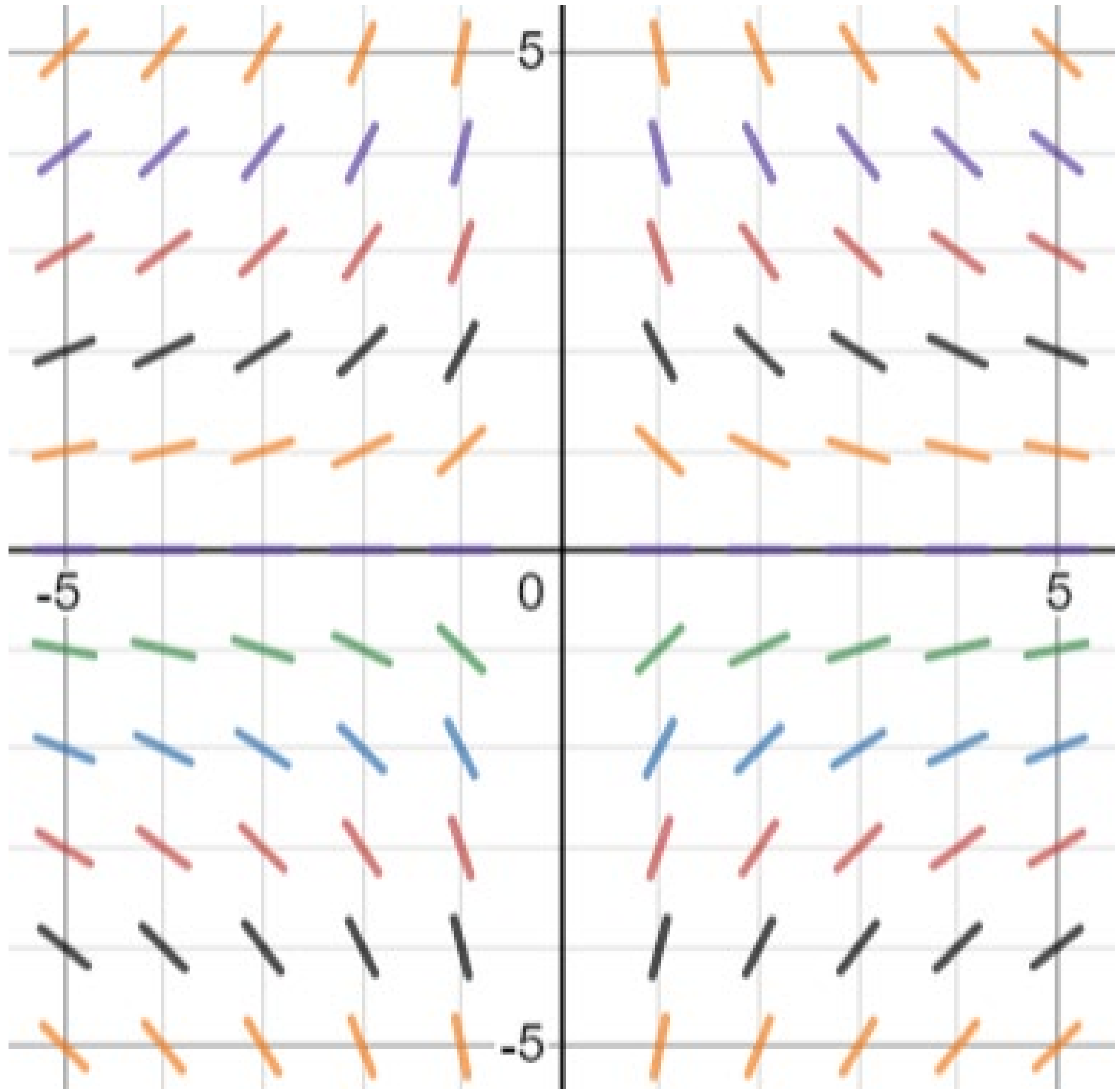
e.g. (*i*) $\frac{dy}{dx} = -\frac{y}{x}$

1. calculate the slope for each point in the grid

2. at each point draw a short interval of that slope

$y \backslash x$	-5	-4	-3	-2	-1	0	1	2	3	4	5
5	1	$\frac{5}{4}$	$\frac{5}{3}$	$\frac{5}{2}$	5	*	-5	$\frac{-5}{2}$	$\frac{-5}{3}$	$\frac{-5}{4}$	-1
4	$\frac{4}{5}$	1	$\frac{4}{3}$	2	4	*	-4	-2	$\frac{-4}{3}$	-1	$\frac{-4}{5}$
3	$\frac{3}{5}$	$\frac{3}{4}$	1	$\frac{3}{2}$	3	*	-3	$\frac{-3}{2}$	-1	$\frac{-3}{4}$	$\frac{-3}{5}$
2	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	1	2	*	-2	-1	$\frac{-2}{3}$	$\frac{-1}{2}$	$\frac{-2}{5}$
1	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	*	-1	$\frac{-1}{2}$	$\frac{-1}{3}$	$\frac{-1}{4}$	$\frac{-1}{5}$
0	0	0	0	0	0	*	0	0	0	0	0
-1	$\frac{-1}{5}$	$\frac{-1}{4}$	$\frac{-1}{3}$	$\frac{-1}{2}$	-1	*	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
-2	$\frac{-2}{5}$	$\frac{-1}{2}$	$\frac{-2}{3}$	-1	-2	*	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$
-3	$\frac{-3}{5}$	$\frac{-3}{4}$	-1	$\frac{-3}{2}$	-3	*	3	$\frac{3}{2}$	1	$\frac{3}{4}$	$\frac{3}{5}$
-4	$\frac{-4}{5}$	-1	$\frac{-4}{3}$	-2	-4	*	4	2	$\frac{4}{3}$	1	$\frac{4}{5}$
-5	-1	$\frac{-5}{4}$	$\frac{-5}{3}$	$\frac{-5}{2}$	-5	*	5	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	1

The slope field would suggest that the general solution is a series of hyperbolas $y = \frac{c}{x}$



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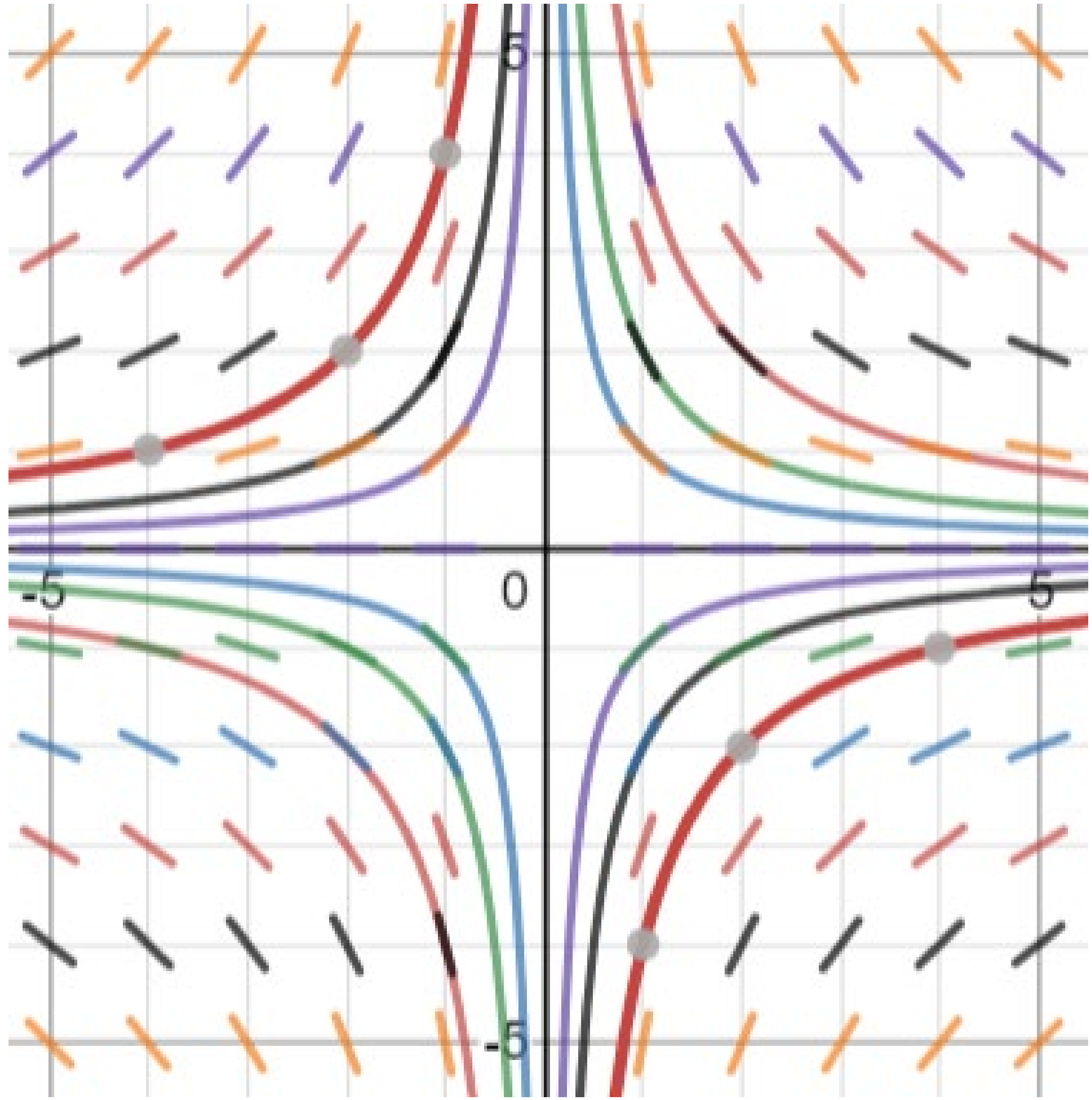
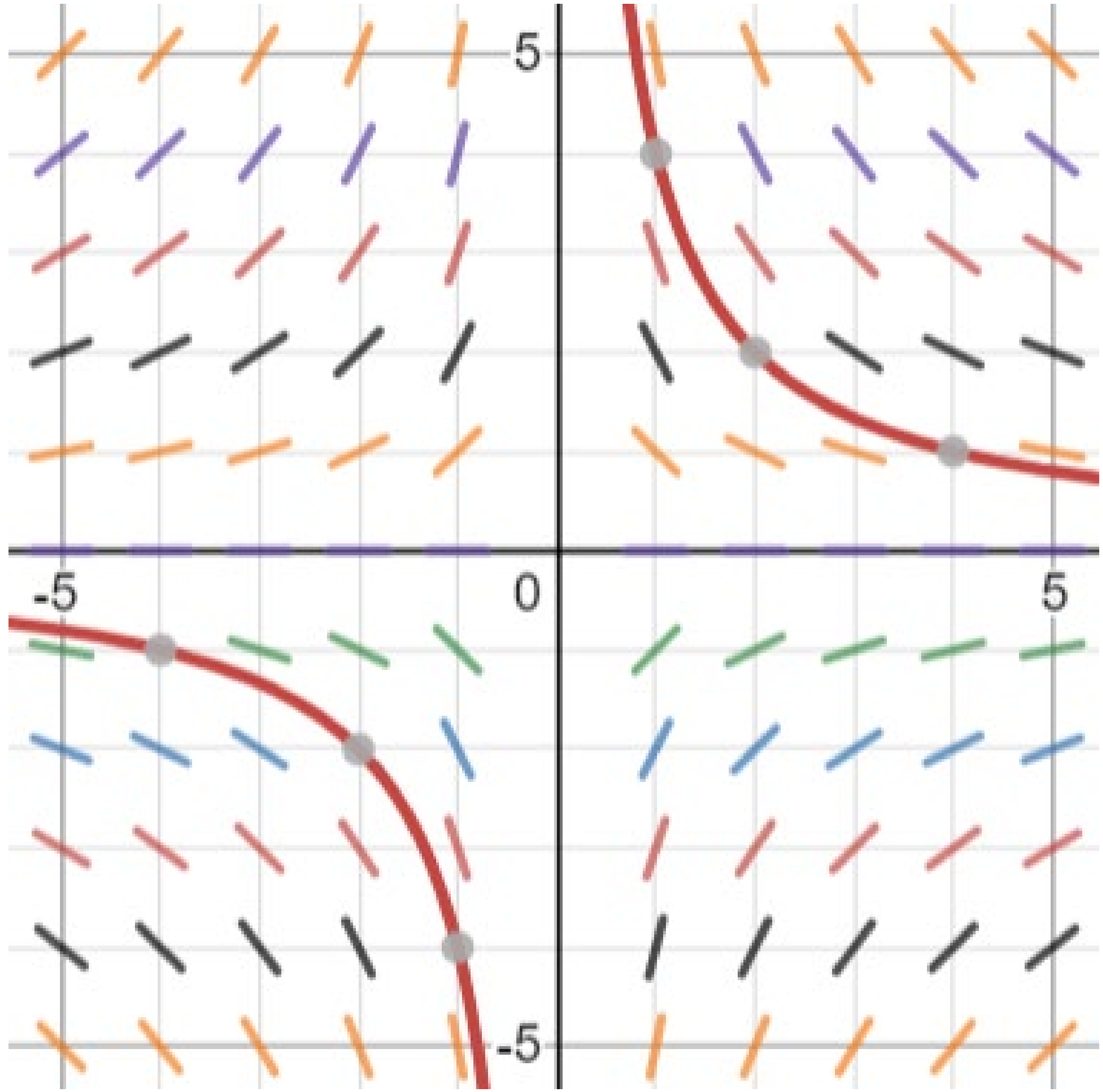


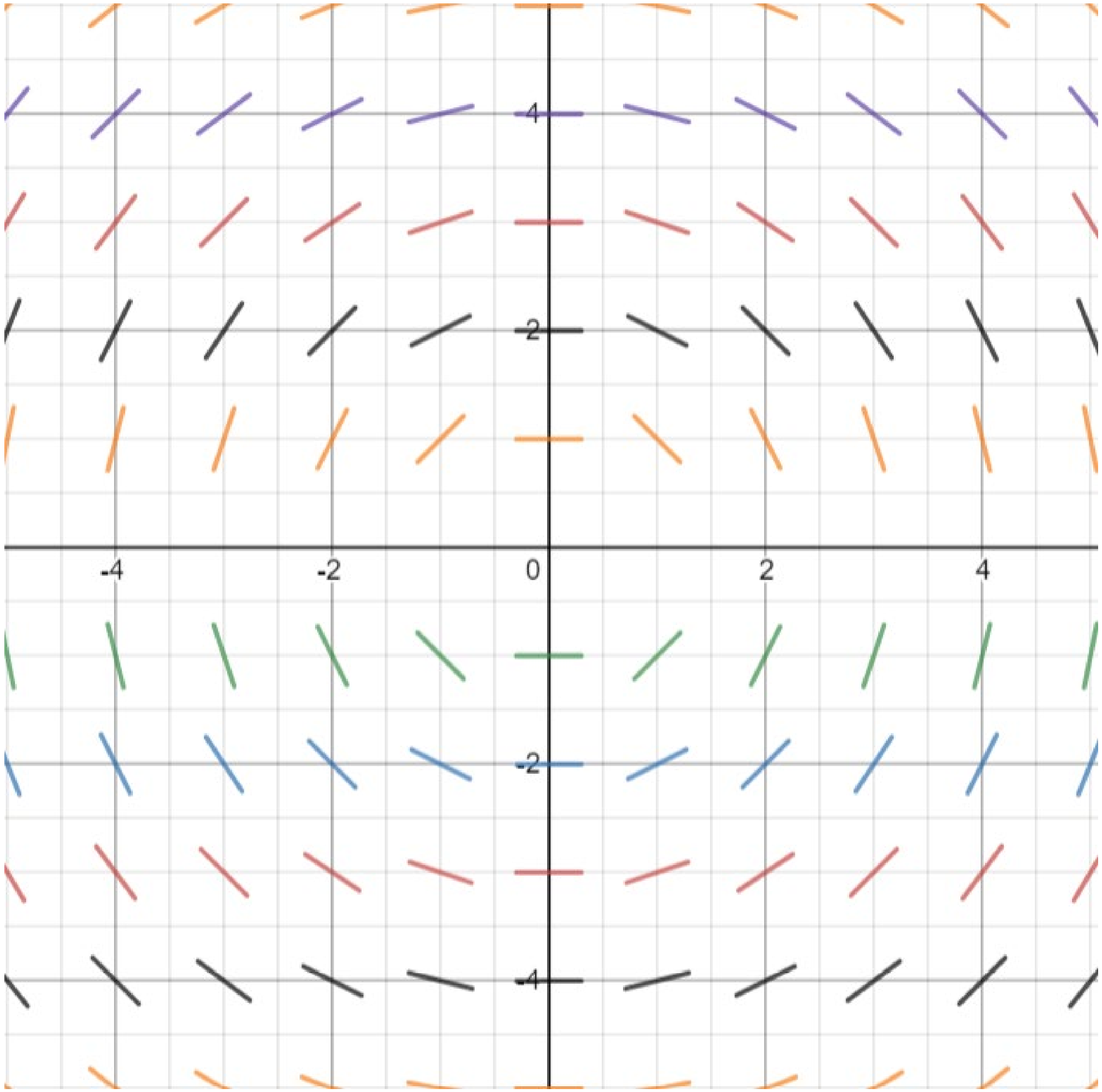
The slope field would suggest that the general solution is a series of hyperbolas $y = \frac{c}{x}$

The particular solution that solves the initial value problem of $y(2) = 2$ would be

$$y = \frac{4}{x}$$



(ii) $\dfrac{dy}{dx} = -\dfrac{x}{y}$



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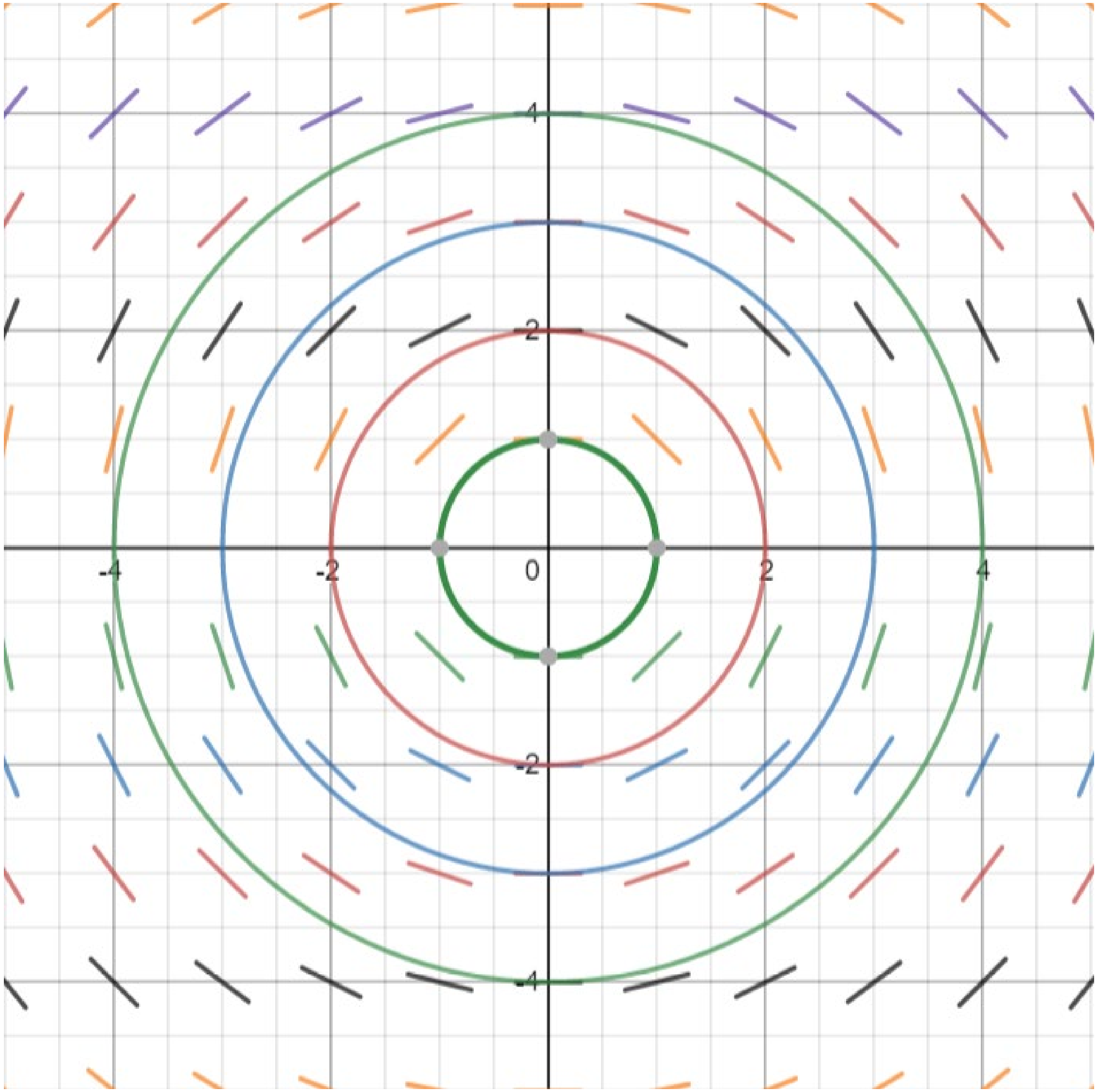
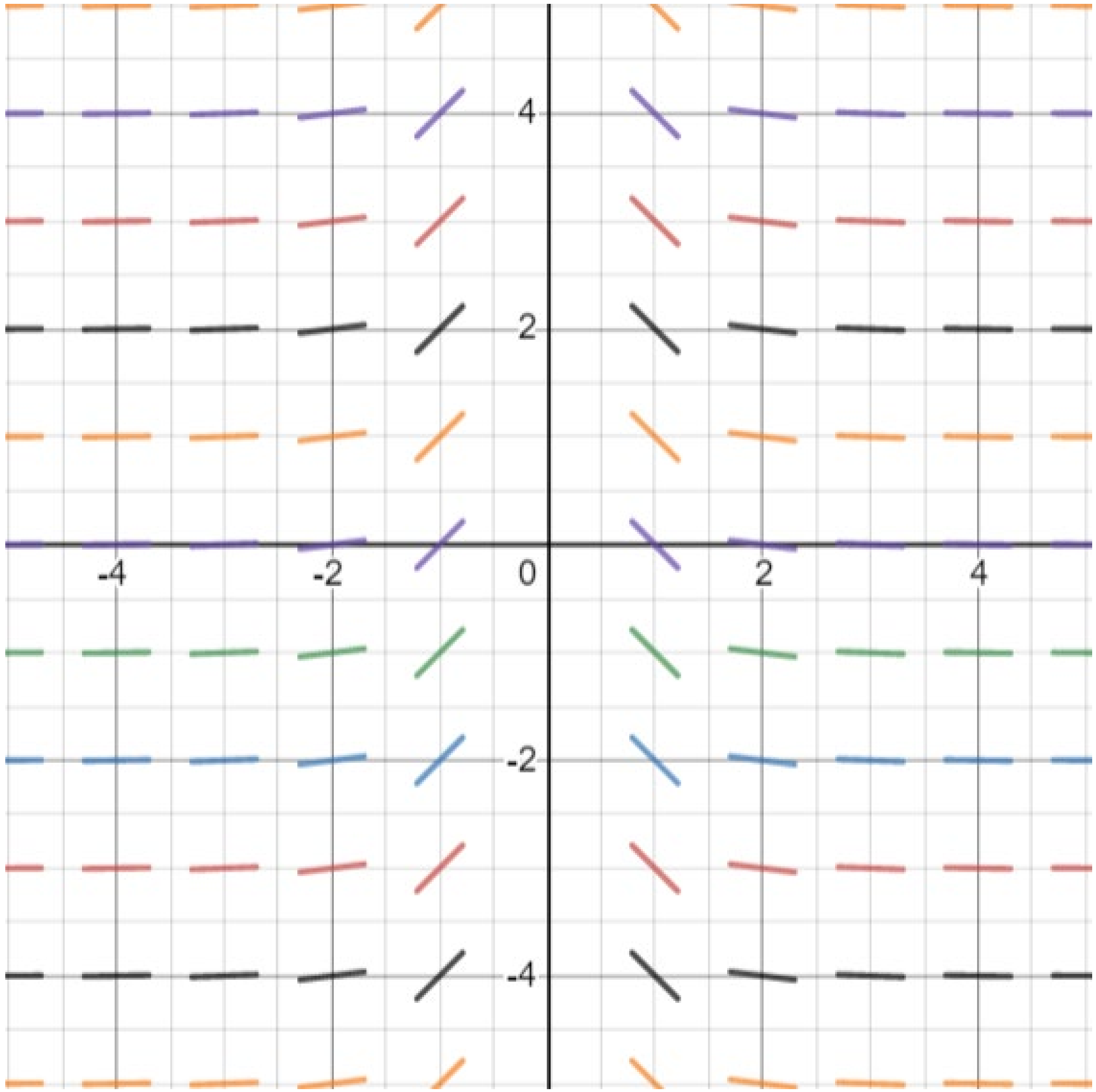


(iii) $\dfrac{dy}{dx} = -\dfrac{1}{x^3}$



if $\frac{dy}{dx} = f(x)$ the slopes appear to be in columns of equal slopes

continual horizontal/vertical lines imply asymptotes

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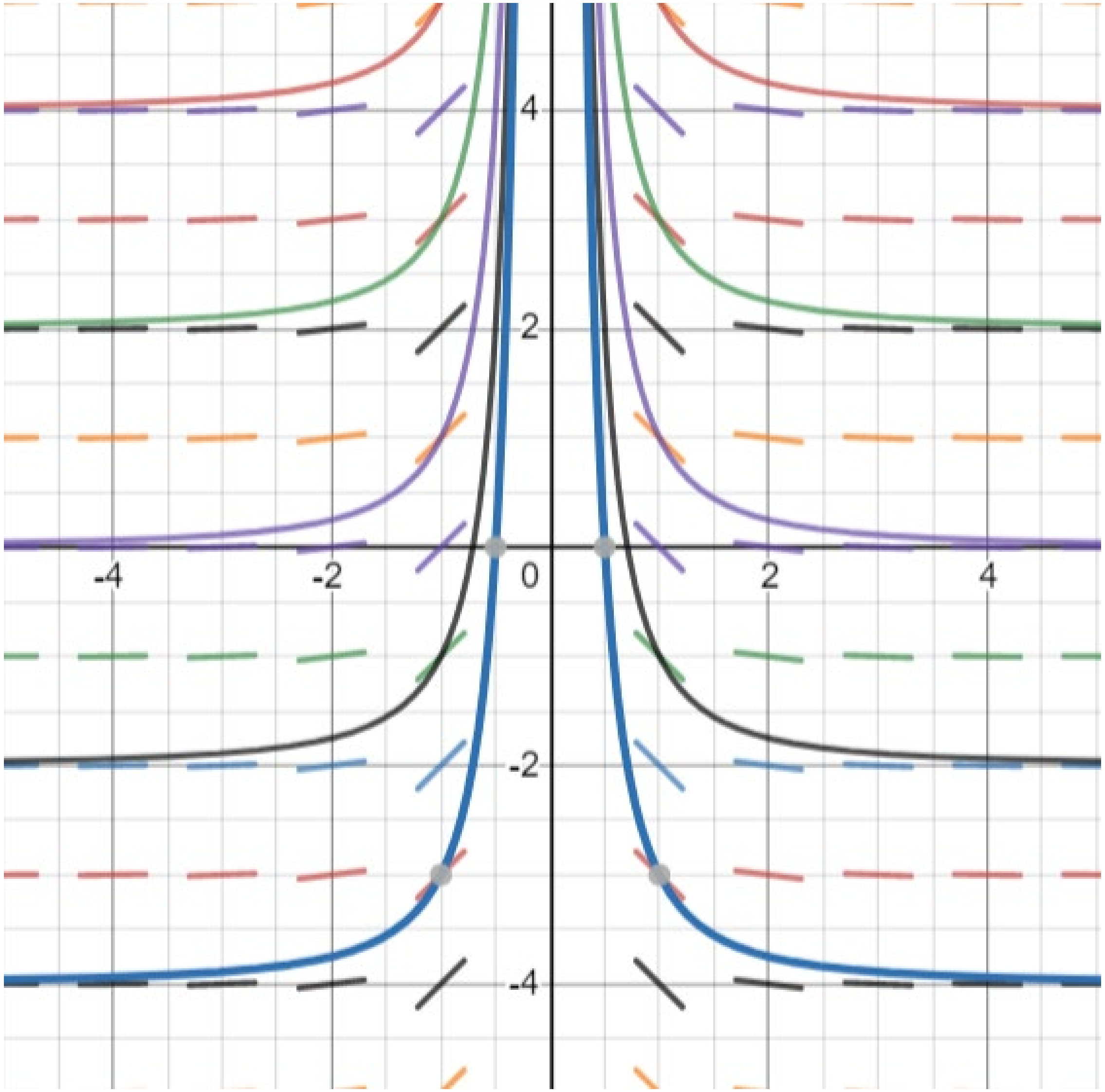

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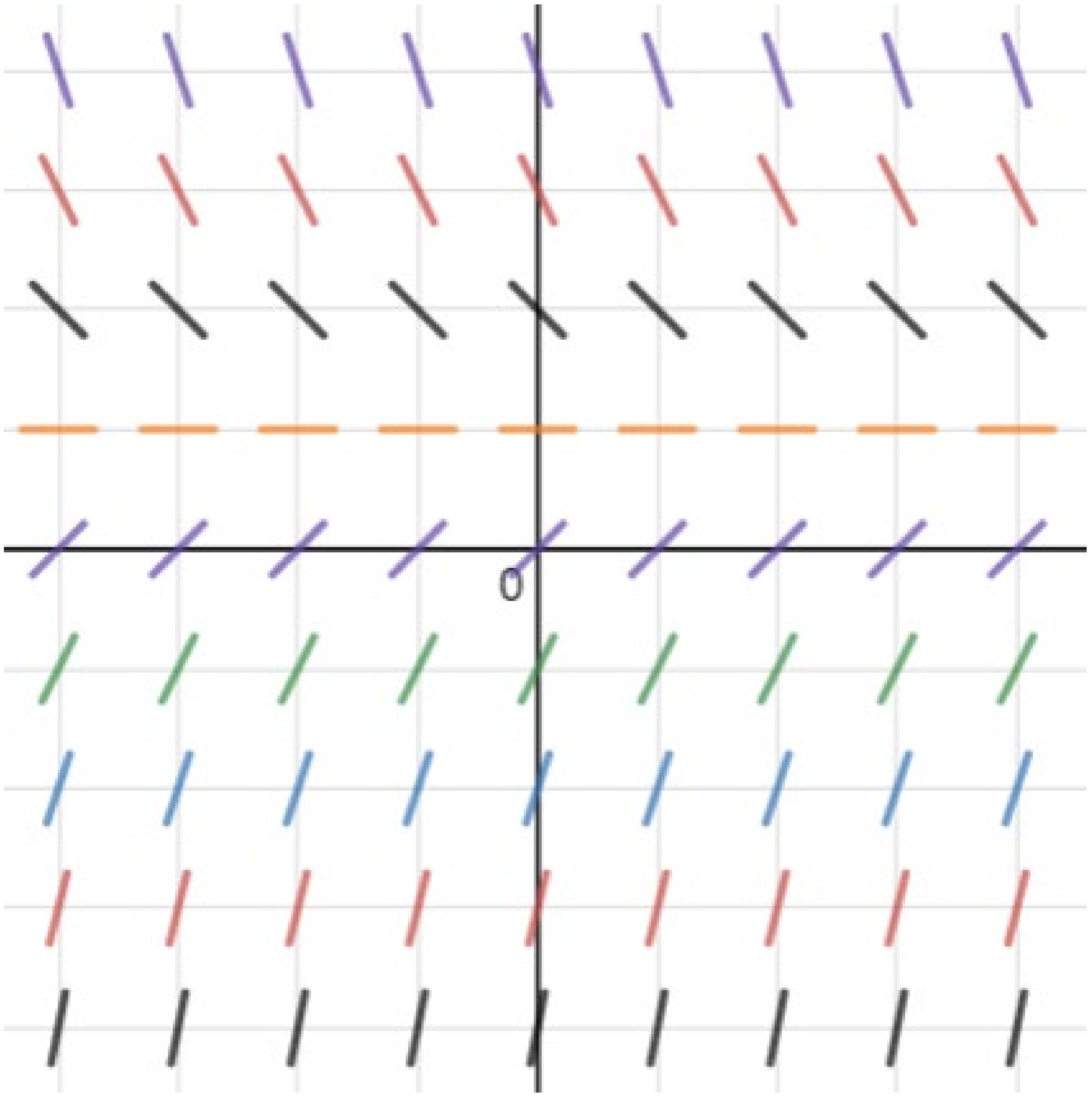
(iv) $\dfrac{dy}{dx} = 1 - y$



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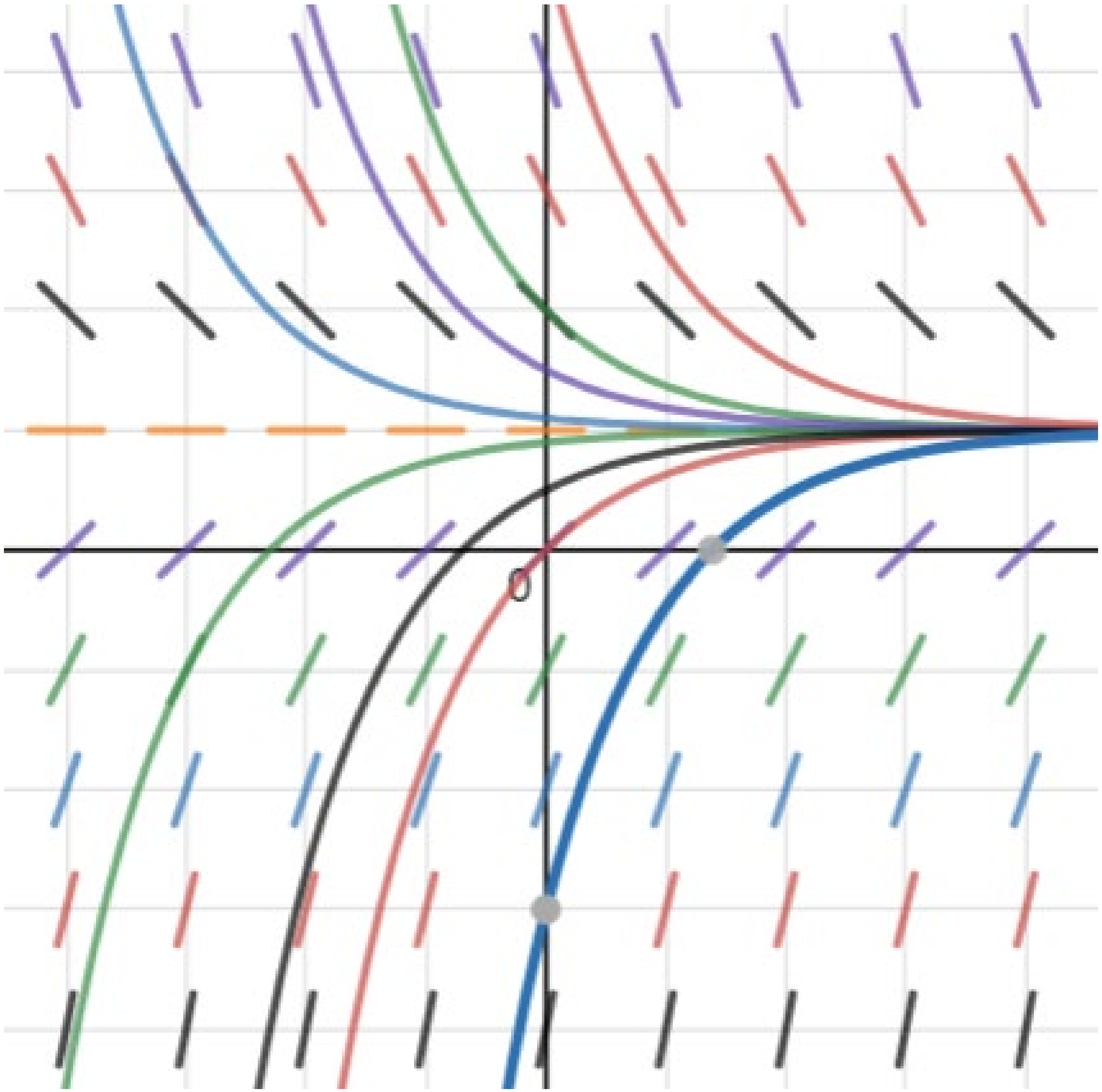

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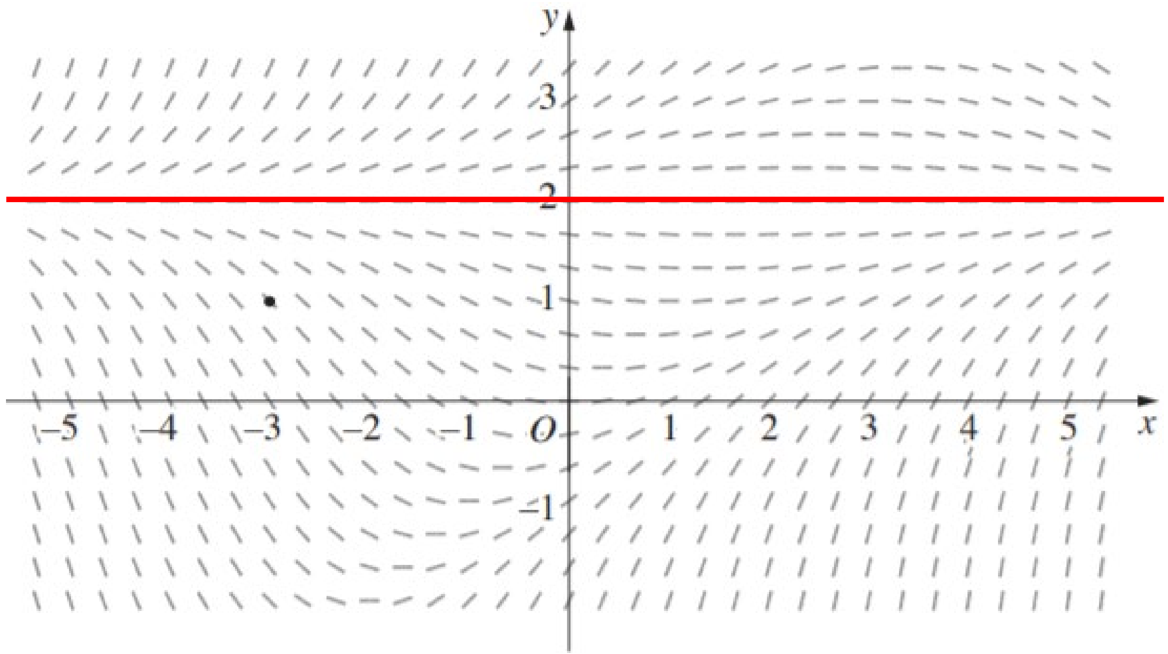
(v) The trajectories of particles in a fluid are described by the differential equation

$$\frac{dy}{dx} = \frac{1}{4}(y - 2)(y - x)$$

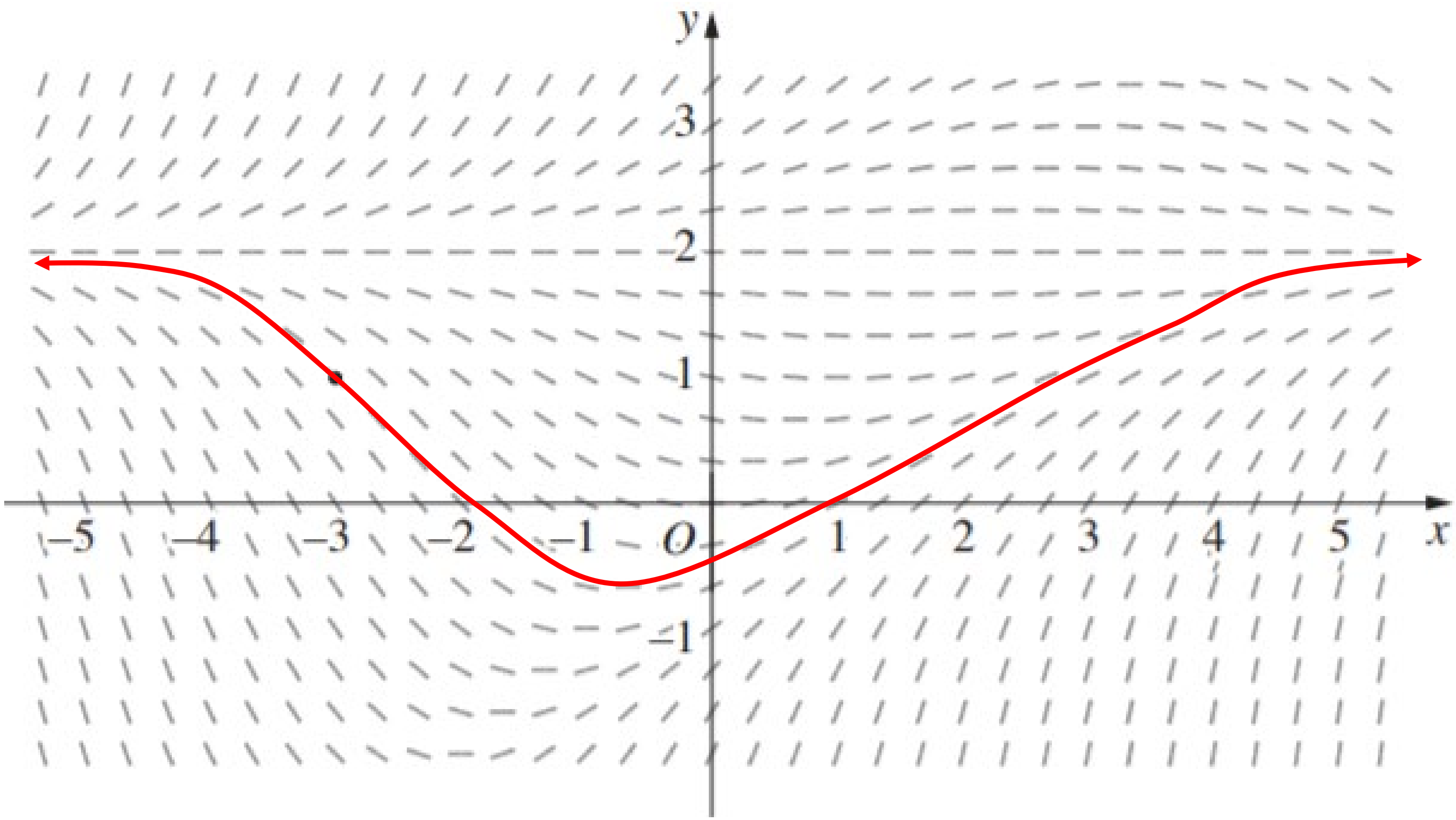
The slope field for the differential equation is shown below



a) Identify any solutions of the form $y = k$, where k is a constant

$y = 2$

b) Draw a sketch of the trajectory of a particle in the fluid which passes through the point $(-3,1)$ and describe the trajectory as $x \to \pm\infty$



as $x \to \pm\infty$, $y \to 2$ from underneath the asymptote

Exercise 13B; 1ef, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15, 16