Vectors in 3D

Unit Vectors

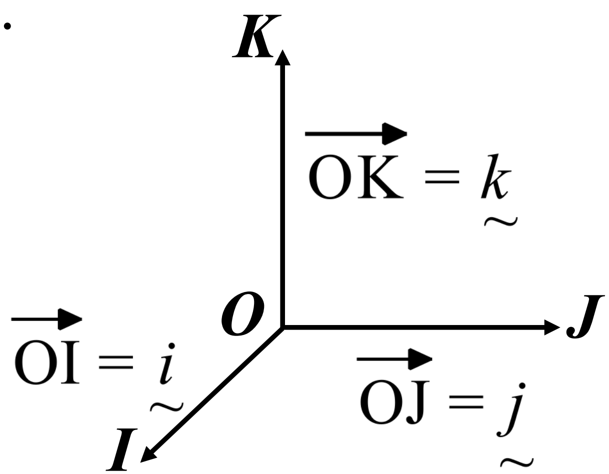
Every non-zero vector has a corresponding unit vector with the same direction

$$\hat{\underset{\sim}{a}} = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|} \quad \text{and} \quad \left|\hat{\underset{\sim}{a}}\right| = 1$$

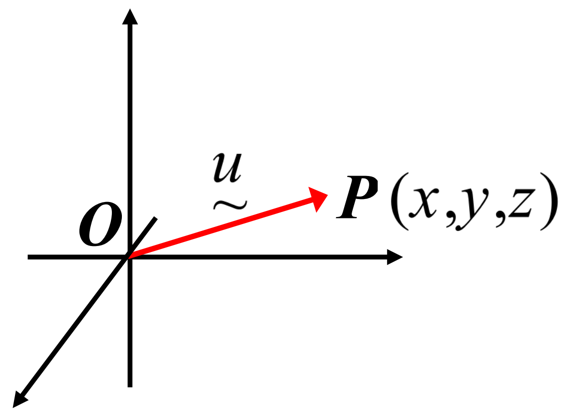
Three Special Unit Vectors

All vectors can be rewritten in terms of components, three special unit vectors that are **orthogonal** (mutually perpendicular).

For convenience we will define them to be in the same orientation as the Cartesian space.



Component Form of a Position Vector



$$\underset{\sim}{u} = \overrightarrow{OP} = (x,y,z) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\,\underset{\sim}{i} + y\,\underset{\sim}{j} + z\,\underset{\sim}{k}$$

position vector	***ordered triple***	***column vector***	***component form***

$$|\underset{\sim}{u}| = \sqrt{x^2 + y^2 + z^2}$$

e.g. $ABCD$ is a parallelogram. The coordinates of A, B and D are (4, 2, 3) , (18, 4, 8) and (– 1, 12, 13) respectively.

a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$

$$\overrightarrow{AB} = \begin{pmatrix} 18 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 2 \\ 5 \end{pmatrix}$$

$$\overrightarrow{\mathrm{AD}} = \begin{pmatrix} -1 \\ 12 \\ 13 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 10 \end{pmatrix}$$

b) Find the coordinates of C

$ABCD$ is a parallelogram

$$\therefore \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$$

$$C - D = B - A$$

$$C = B - A + D$$

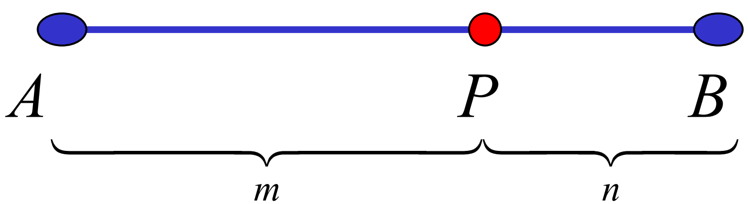
$$C = \begin{pmatrix} 18 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 12 \\ 13 \end{pmatrix} = \begin{pmatrix} 13 \\ 14 \\ 18 \end{pmatrix}$$

$\therefore C$ is (13,14,18)

Division Of An Interval

Midpoint is dividing an interval in the ratio 1:1

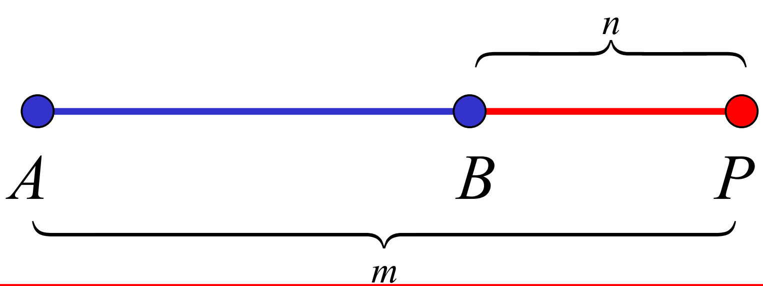
You can of course, divide an interval in a any ratio, and it could be either an internal or an external division.



P divides *AB* internally in the ratio *m*:*n*

OR

P divides *BA* internally in the ratio *n*:*m*



P divides *AB* externally in the ratio *m*:*n*

If *P* divides *AB* in the ratio *m*:*n*, then;

$$\underset{\sim}{p} = \frac{1}{m+n}\left(n\underset{\sim}{a} + m\underset{\sim}{b}\right)$$

where $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{p}$ are the position vectors of $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OP}$

Type 1: Internal Division

Find the coordinates of P that divides the interval joining $\begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}$ internally in the ratio 1 : 3

$$\underset{\sim}{p} = \frac{3}{4}\begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ \frac{9}{2} \\ \frac{3}{2} \end{pmatrix}$$

$$\therefore P \text{ is } \left(1, \frac{9}{2}, \frac{3}{2}\right)$$

Type 2: External Division (*negative ratio*)

Let $\underset{\sim}{a} = \begin{pmatrix} 3 \\ -1 \\ 6 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 9 \\ 2 \\ -3 \end{pmatrix}$

Find the position vector that divides *AB* externally in the ratio 5 : 2.

(Done exactly the same as internal division, except make one of the numbers in the ratio negative)

$$\underset{\sim}{p} = -\frac{2}{3}\begin{pmatrix} 3 \\ -1 \\ 6 \end{pmatrix} + \frac{5}{3}\begin{pmatrix} 9 \\ 2 \\ -3 \end{pmatrix}$$

$$\underset{\sim}{p} = \begin{pmatrix} 13 \\ 4 \\ -9 \end{pmatrix}$$

Divide externally in the ratio 5 : 2
is the same as
divide
in the ratio 5: – 2

Exercise 5B; 1a, 2b, 4ab, 6, 8, 10, 12, 14, 15, 16, 17, 18