

Differentiating Non-Cardinal Powers

$$(1) \quad y = \frac{1}{x}$$

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - x - h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$$

Note:

$$f(x) = x^{-1}$$

$$\begin{aligned} f'(x) &= -x^{-2} \\ &= \frac{-1}{x^2} \end{aligned}$$

(2) $y = \sqrt{x}$

$$f(x) = \sqrt{x}$$

$$\begin{aligned} f(x+h) &= \sqrt{x+h} & f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ && &= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\ && &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ && &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ && &= \frac{1}{\sqrt{x} + \sqrt{x}} \\ && &= \frac{1}{2\sqrt{x}} \end{aligned}$$

Note:

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\begin{aligned}\text{e.g. } (i) \quad &y = 3x + \frac{1}{x^2} \\&= 3x + x^{-2} \\&\frac{dy}{dx} = 3 - 2x^{-3} \\&= 3 - \frac{2}{x^3} \\&\underline{\hspace{2cm}}\end{aligned}$$

$$\begin{aligned}(iii) \quad &y = \frac{2}{(5x+4)^2} \\&= 2(5x+4)^{-2} \\&\frac{dy}{dx} = -4(5x+4)^{-3}(5) \\&= -20(5x+4)^{-3} \\&= \frac{-20}{(5x+4)^3} \\&\underline{\hspace{2cm}}\end{aligned}$$

$$\begin{aligned}(ii) \quad &y = x^2 \sqrt{x} \\&= x^{\frac{5}{2}} \\&\frac{dy}{dx} = \frac{5}{2} x^{\frac{3}{2}} \\&= \frac{5}{2} x \sqrt{x} \\&\underline{\hspace{2cm}}\end{aligned}$$

$$\begin{aligned}(iv) \quad &y = \sqrt{x^2 - 3} \\&= (x^2 - 3)^{\frac{1}{2}} \\&\frac{dy}{dx} = \frac{1}{2} (x^2 - 3)^{-\frac{1}{2}} (2x) \\&= x (x^2 - 3)^{-\frac{1}{2}} \\&= \frac{x}{\sqrt{x^2 - 3}} \\&\underline{\hspace{2cm}}\end{aligned}$$

Exercise 9F; 1ace, 2bd, 3ad, 5a, 6b, 8a, 9b, 11, 14, 15

Exercise 9G; 1bdf, 2c iii, 4bd, 5b, 6b, 8a, 9c, 10ad, 13, 15