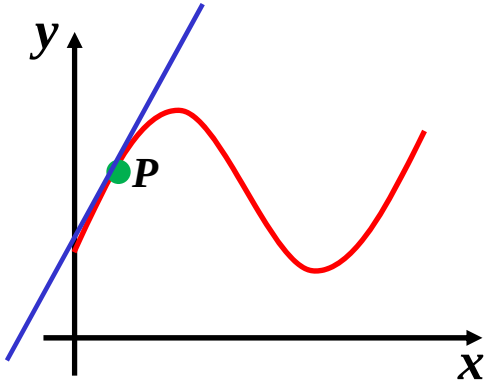
Rates of Change

A derivative measures the rate of one quantity changing with respect to another quantity

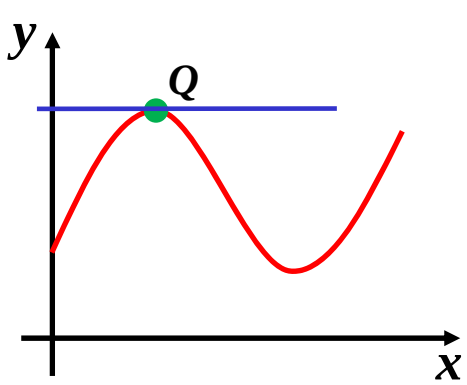
The steeper the curve, the faster the quantity changes



at P, $\dfrac{dy}{dx} > 0$

$f(x)$ is

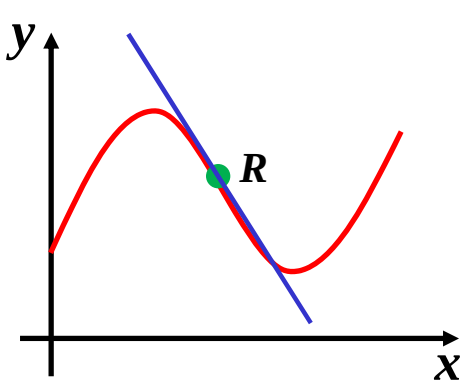
increasing



at Q, $\dfrac{dy}{dx} = 0$

$f(x)$ is

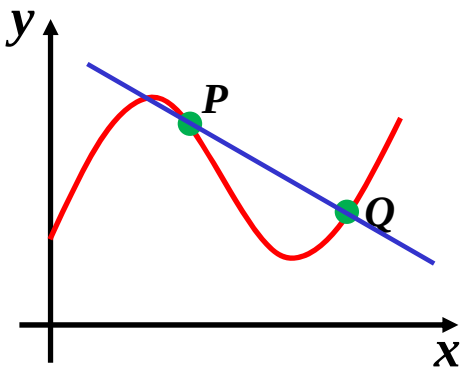
stationary



at R, $\dfrac{dy}{dx} < 0$

$f(x)$ is

decreasing

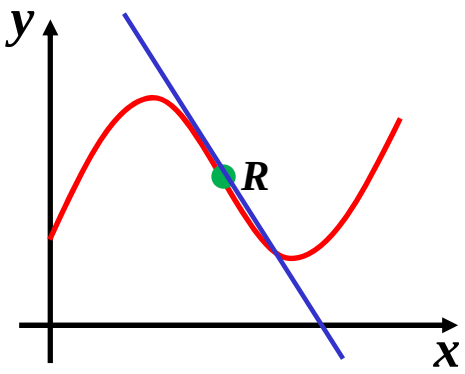


the slope of the secant PQ

gives the

average

rate of change



the slope of the tangent at R

gives the

instantaneous

rate of change

e.g. A water tank is being emptied. The quantity Q litres of water remaining in the tank at any time t minutes after it starts to empty is given by $Q = 1000(20 - t)^2$

a) At what rate is the tank being emptied at any time t?

$$Q = 1000(20 - t)^2$$

$$\frac{dQ}{dt} = 2000(20 - t)^1(-1)$$

$$= -2000(20 - t) \text{ L/min}$$

b) How much time does it take to empty the tank?

Tank will be empty when $Q = 0$

$$\text{i.e. } 1000(20 - t)^2 = 0$$

$$t = 20$$

$\therefore$the tank will be empty after 20 minutes

c) At what time is the water flowing out at a rate of 20000 L/min?

$$\frac{dQ}{dt} = -20000$$

$$-2000(20-t) = -20000$$

$$20-t = 10$$

$$t = 10$$

$\therefore$the tank is emptying at a rate of 20000 L/min after 10 minutes

d) What is the average rate at which the water flows out in the first 5 minutes?

when $t = 0; Q = 1000(20-0)^2$

$= 400000$

when $t = 5; Q = 1000(20-5)^2$

$= 225000$

$$\text{average rate} = \frac{225000 - 400000}{5-0}$$

$$= -35000$$

$\therefore$the tank is emptying at an average rate of 35000 L/min in the first 5 minutes

Equations of Motion

Calculus was discovered independently by two mathematicians;

Gottfried Leibniz (Germany 1674 – aged 28).
It is Leibniz's notations that have been adopted for modern calculus.

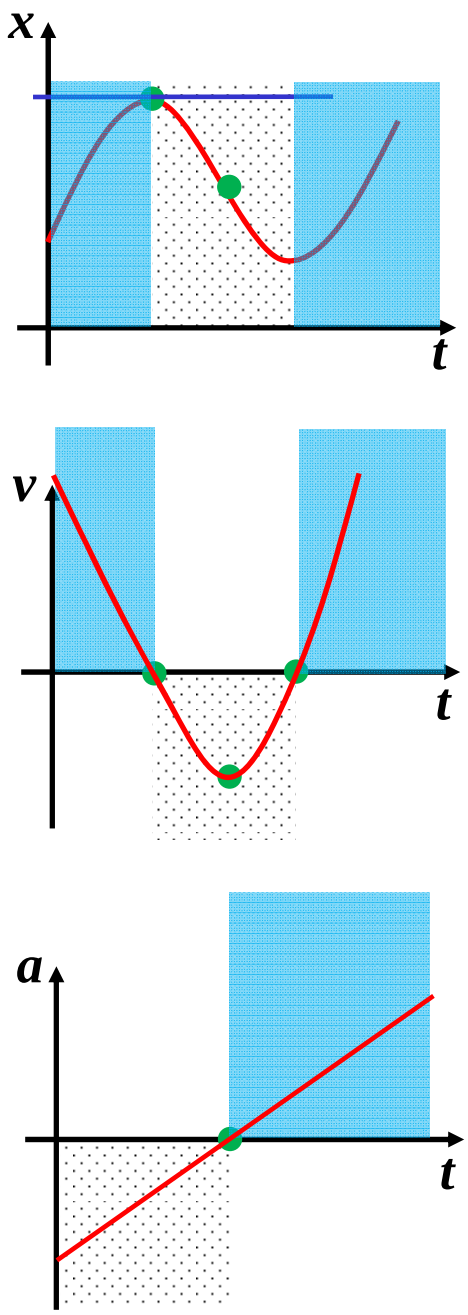
Issac Newton (England 1665 – aged 22)
Newton used calculus to explain his laws of motion. Newton's dot notation is used when describing motion.

Displacement, Velocity & Acceleration

Displacement (x) Distance from a point, with direction.

Velocity $\left(v, \dot{x}, \frac{dx}{dt} \right)$ The rate of change of displacement with respect to time i.e. speed with direction.

Acceleration $\left(a, \dot{v}, \ddot{x}, \frac{dv}{dt}, \frac{d^2x}{dt^2} \right)$ The rate of change of velocity with respect to time



When displacement graph is stationary, velocity is zero

When displacement graph is increasing, velocity is positive

Similarly, when displacement graph is decreasing, velocity is negative

Velocity graph is stationary, when displacement graph inflects

Similar relationships exist between the velocity and acceleration graphs

Exercise 9J; 2, 3, 4, 6, 7, 8, 10, 12, 13, 14, 15