Differentiating Exponentials

$$y = e^{f(x)}$$

$$\text{let } u = f(x)$$

$$y = e^{u}$$

$$\frac{dy}{dx} = \frac{d(e^{u})}{du} \times \frac{du}{dx}$$

$$= e^{u} \times f'(x)$$

$$= f'(x)e^{f(x)}$$

$y = e^{f(x)}$ $\frac{dy}{dx} = f'(x)e^{f(x)}$	$y = a^{f(x)}$ $\frac{dy}{dx} = f'(x)(\log a)a^{f(x)}$

e.g. $(i)\ y=e^{x}$

$$\frac{dy}{dx}=e^{x}$$

$(ii)\ y=e^{5x}$

$$\frac{dy}{dx}=5e^{5x}$$

$(iii)\ y=e^{4x+3}$

$$\frac{dy}{dx}=4e^{4x+3}$$

$(iv)\ y=e^{x^2+3x+2}$

$$\frac{dy}{dx}=(2x+3)e^{x^2+3x+2}$$

(v) Find the tangent to $y = e^{2x} + 1$ at the point $(1, e^2 + 1)$

$$y=e^{2x}+1$$

$$\frac{dy}{dx}=2e^{2x}$$

when $x=1, \frac{dy}{dx}=2e^{2}$

$$y-(e^2+1)=2e^2(x-1)$$

$$y-e^2-1=2e^2x-2e^2$$

$$2e^2x-y-e^2+1=0$$

$(vi)\ y = 4^{x^2}$

$$\frac{dy}{dx}=2x(\log 4)4^{x^2}$$

The Natural Logarithm

The inverse function to the exponential function, base *e*, is the **natural logarithm**

$$\text{If } y = e^x \quad \text{then } x = \log_e y$$
$$x = \ln y$$
$$x = \log y$$

e.g. Simplify

$(i)\ \ln\frac{1}{e} = \ln e^{-1}$

$= -\ln e$

$= -1$

$(ii)\ e^{2\ln 3} = e^{\ln(3^2)}$

$= 3^2$

$= 9$

**Exercise 11C; 1adfh, 2bh, 3ace,
4, 6a, 7bd, 8de, 9ad, 11b, 12**

**Exercise 11D; 1, 4, 5a, 6, 8, 10,
11b, 14, 15, 16**

Exercise 11E; 1bg, 2b, 3e, 5bij, 6c, 8efhi, 9d, 10f, 11ac, 13ad