Double Angles

$$\sin 2\theta = \sin(\theta + \theta)$$

$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\cos 2\theta = \cos(\theta + \theta)$$

$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$= \cos^2\theta - \left(1 - \cos^2\theta\right)$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$= 2\left(1 - \sin^2\theta\right) - 1$$

$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

Double Angles

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

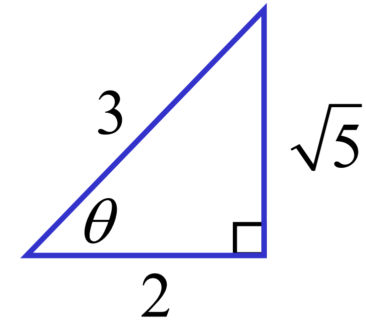
$$= 2\cos^2\theta - 1 \qquad \Rightarrow \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$= 1 - 2\sin^2\theta \qquad \Rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

e.g. (i) If $\cos\theta = \frac{2}{3}$, find $\tan 2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$



$$\tan 2\theta = \frac{2\left(\frac{\sqrt{5}}{2}\right)}{1-\left(\frac{\sqrt{5}}{2}\right)^2}$$

$$= \frac{\sqrt{5}}{-\frac{1}{4}}$$

$$= -4\sqrt{5}$$

(ii) Find the exact value of $\sin\frac{5\pi}{12}\cos\frac{5\pi}{12}$

$$\sin\frac{5\pi}{12}\cos\frac{5\pi}{12} = \frac{1}{2}\left(2\sin\frac{5\pi}{12}\cos\frac{5\pi}{12}\right)$$

$$= \frac{1}{2}\sin\left(2\times\frac{5\pi}{12}\right)$$

$$= \frac{1}{2}\sin\frac{5\pi}{6}$$

$$= \frac{1}{2}\times\frac{1}{2}$$

$$= \frac{1}{4}$$

(iii) If $\cos\theta = \frac{2}{3}$, find the exact value of $\sin\frac{\theta}{2}$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\therefore \sin^2\frac{\theta}{2} = \frac{1}{2}(1-\cos\theta)$$

$$= \frac{1}{2}\left(1-\frac{2}{3}\right)$$

$$= \frac{1}{6}$$

$$\sin\frac{\theta}{2} = \pm\frac{1}{\sqrt{6}}$$

(iv) Prove $\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}} \equiv \tan x$

$$\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} = \sqrt{\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}}$$

$$= \sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$$

$$= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \sqrt{\tan^2 x}$$

$$= \tan x$$

(v) Prove that $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$ 1996 Extension 1 HSC Q4a)

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin(3\theta - \theta)}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta}{\sin 2\theta}$$

$$= 2$$

(vi) Prove the following identity; 1994 Extension 1 HSC Q2a)

$$\frac{2\tan A}{1+\tan^2 A} = \sin 2A$$

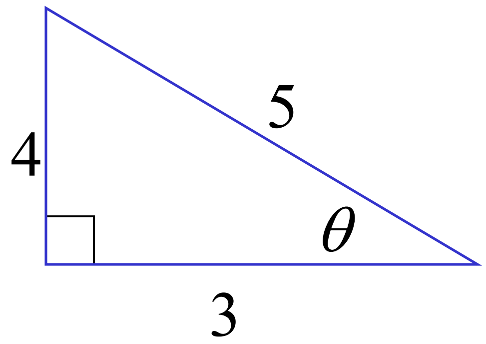
$$\frac{2\tan A}{1+\tan^2 A} = \frac{\frac{2\sin A}{\cos A}}{1+\frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{2\sin A\cos A}{\cos^2 A+\sin^2 A}$$

$$= \frac{\sin 2A}{1}$$

$$= \sin 2A$$

$(vii)\ \sin\left(2\cos^{-1}\frac{3}{5}\right)$ let $\theta = \cos^{-1}\frac{3}{5}$



$= 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{24}{25}$

$(viii)\ \cos^{-1}\left(2\cos\frac{\pi}{3}\right) = \cos^{-1}\left(2\times\frac{1}{2}\right)$

$= \cos^{-1} 1$

$= 0$

Exercise 17E; 1def, 2cd, 3bd, 4, 5acf, 6, 7b, 8bc, 9, 10a, 12, 13, 14bde, 15, 16, 17