Translating Curves

Horizontal Shift replace x with $x - h$

$y = f(x-h)$

moves $f(x)$ $\underset{-}{\Leftarrow}$ or $\underset{+}{\Rightarrow}$ h units

to find where the origin has been moved to; solve $x - h = 0$

Vertical Shift replace y with $y - k$

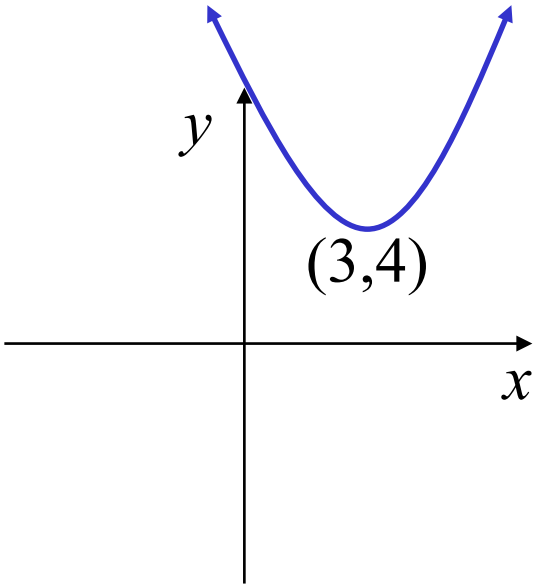
$(y-k) = f(x)$ ***OR*** $y = f(x)+k$

moves $f(x)$ $\underset{+}{\Uparrow}$ or $\underset{-}{\Downarrow}$ k units

to find where the origin has been moved to; solve $y - k = 0$

e.g. $(i)\ y=(x-3)^2+4$

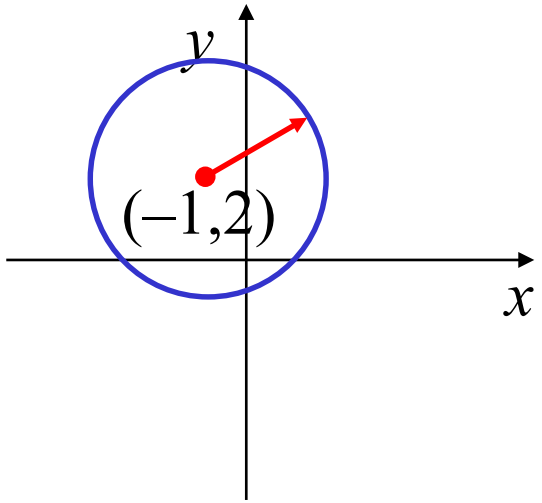
1. *basic curve*: $y=x^2$
2. *shift right 3 units*
3. *shift up 4 units*



$(ii)\ x^2+2x+y^2-4y-4=0$

$$(x+1)^2+(y-2)^2=4+1+4$$
$$=9$$

1. *basic curve*: $x^2+y^2=9$
2. *shift left 1 unit*
3. *shift up 2 units*



$(iii)\ y = 2x^2 + 8x + 5$

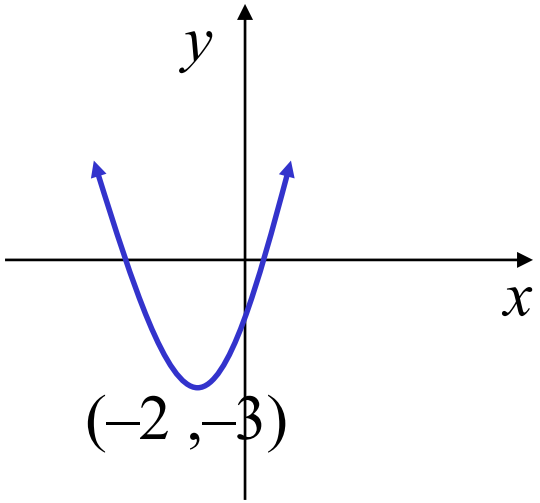
$$\frac{y}{2} = x^2 + 4x + \frac{5}{2}$$

$$\frac{y}{2} = (x+2)^2 + \frac{5}{2} - 4$$

$$= (x+2)^2 - \frac{3}{2}$$

$$y = 2(x+2)^2 - 3$$

It is easier to complete the square of x^2 than $2x^2$



1. *basic curve*: $y = x^2$
2. *curve gets steeper*
3. *shift left 2 units*
4. *shift down 3 units*

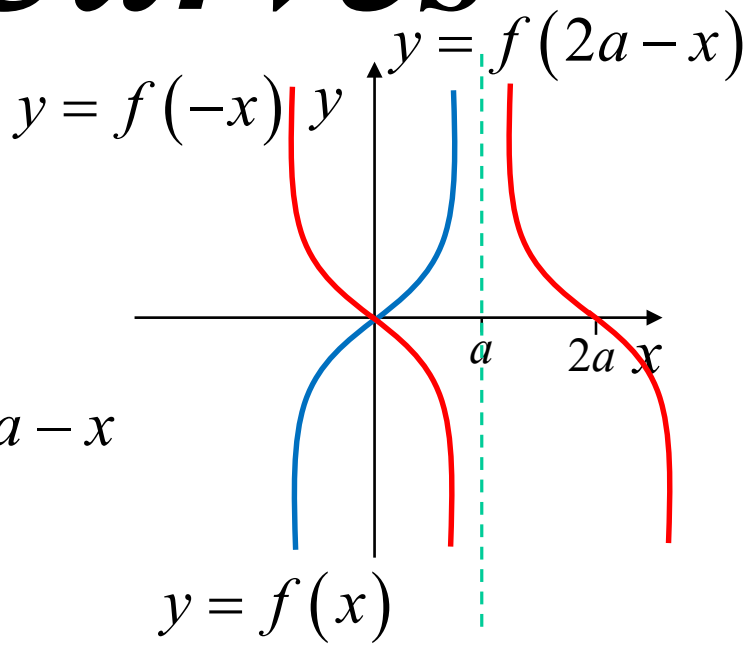
Reflecting Curves

Vertical Reflection replace x with $-x$

$y = f(-x)$

Reflects in the y-axis

To reflect in the line $x = a$; replace x with $2a - x$



Horizontal Reflection replace y with $-y$

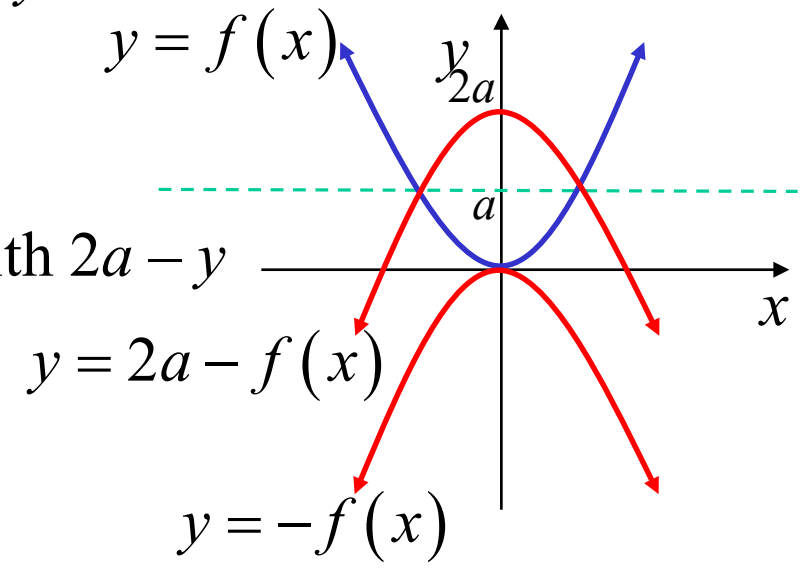
$-y = f(x)$ ***OR*** $y = -f(x)$

Reflects in the x-axis

To reflect in the line $y = a$; replace y with $2a - y$

i.e. $y = 2a - f(x)$

reflection before translation

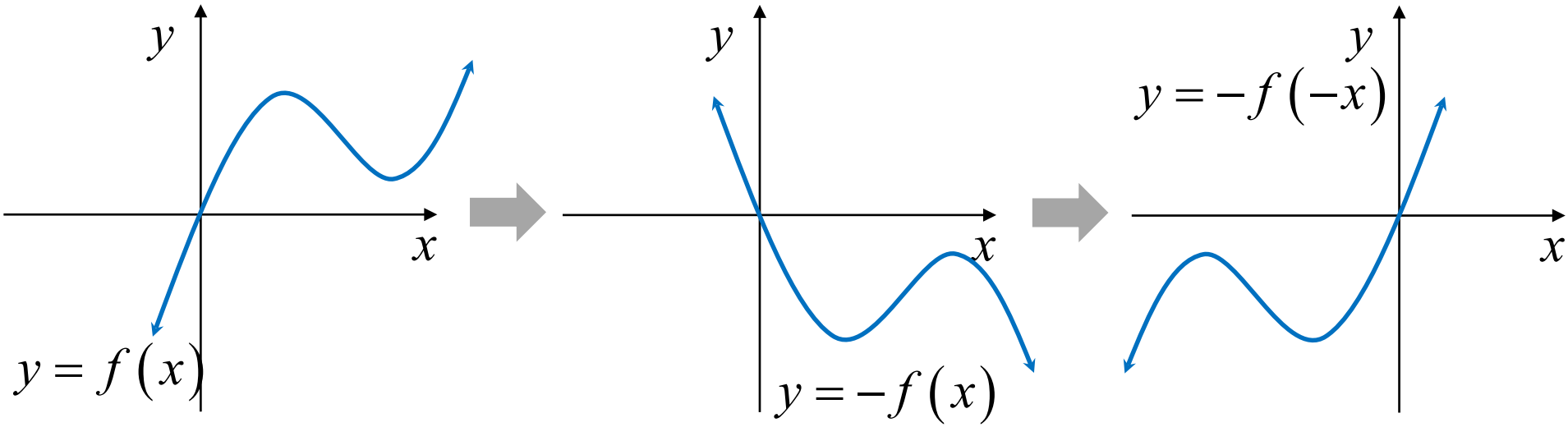


Reflection in the origin

$y = -f(-x)$

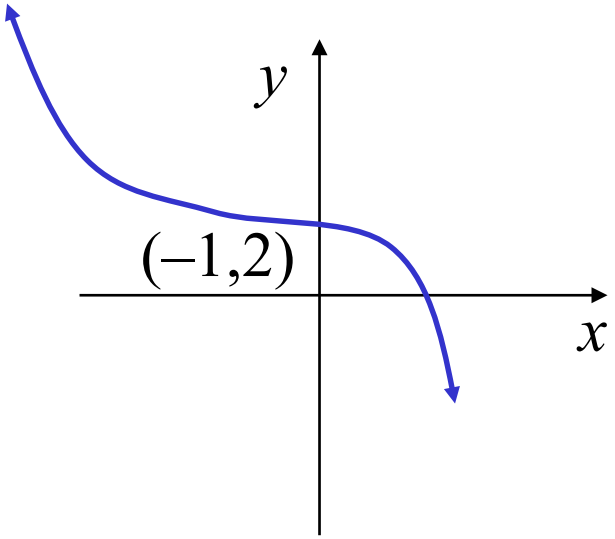
A reflection in the origin is the same as a rotation of 180°

It is achieved by two other reflections, one in the x-axis, the other in the y-axis



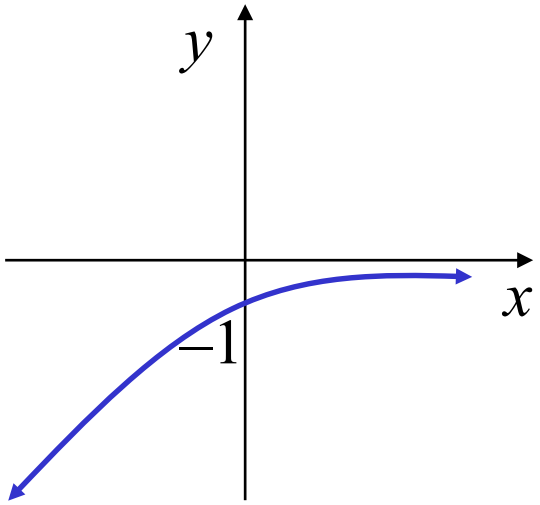
e.g. $(i)\ y = -(x+1)^3 + 2$

1. *basic curve*: $y = x^3$
2. *reflect in x axis*
3. *shift left 1 unit*
4. *shift up 2 units*



$(ii)\ y = -2^{-x}$

1. *basic curve*: $y = 2^x$
2. *reflect in origin*



Exercise 3G; 1aceg, 2aceg, 4d, 5a, 6, 7, 8ace, 10a, 12, 13