De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

for all integers n

this extends to;

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

e.g. $(1-i)^5$

$$= \left[\sqrt{2}cis\left(-\frac{\pi}{4}\right)\right]^5$$

$$= (\sqrt{2})^5 cis\left(-\frac{5\pi}{4}\right)$$

$$= 4\sqrt{2}cis\left(\frac{3\pi}{4}\right)$$

$$|z| = \sqrt{1^2 + (-1)^2}$$

$$= \sqrt{2}$$

$$\arg z = \tan^{-1}\left(\frac{-1}{1}\right)$$

$$= -\frac{\pi}{4}$$

$$(1-i)^5$$

$$= 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= -4 + 4i$$

Finding Roots

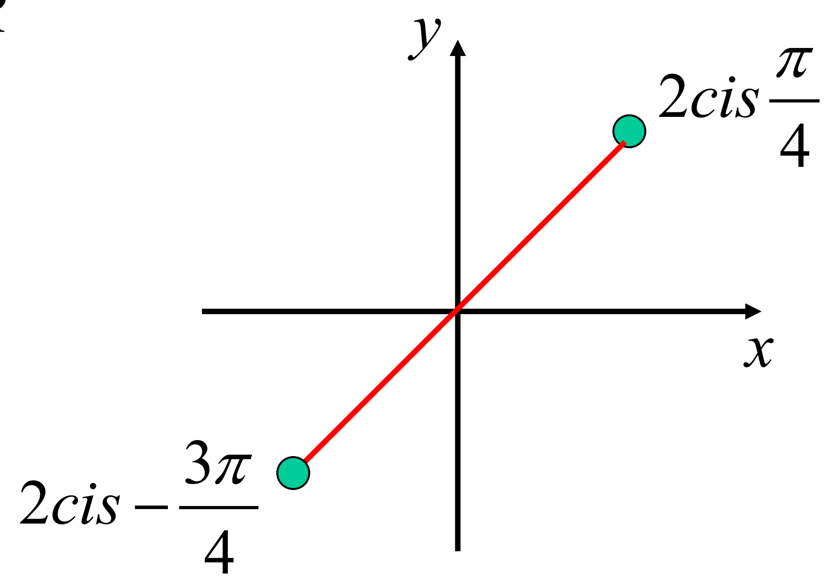
$$\text{If } z^n = x + iy$$
$$z^n = r\,cis\,\theta$$
$$z = \sqrt[n]{r}\,cis\left[\frac{2\pi k + \theta}{n}\right] \qquad k = 0, 1, \ldots, n-1$$

If solutions are in conjugate pairs use 0, ±1, ±2, etc

OR

e.g. (i) $z^2 = 4i$

$$z^2 = 4cis\frac{\pi}{2}$$
$$z = 2cis\left[\frac{2\pi k + \frac{\pi}{2}}{2}\right] \qquad k = 0, 1$$
$$z = 2cis\frac{\pi}{4}, 2cis\frac{5\pi}{4}$$
$$z = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right), 2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$$



$$z = \sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i$$

$(ii)\ x^4 - 16 = 0$

$$x^4 = 16$$

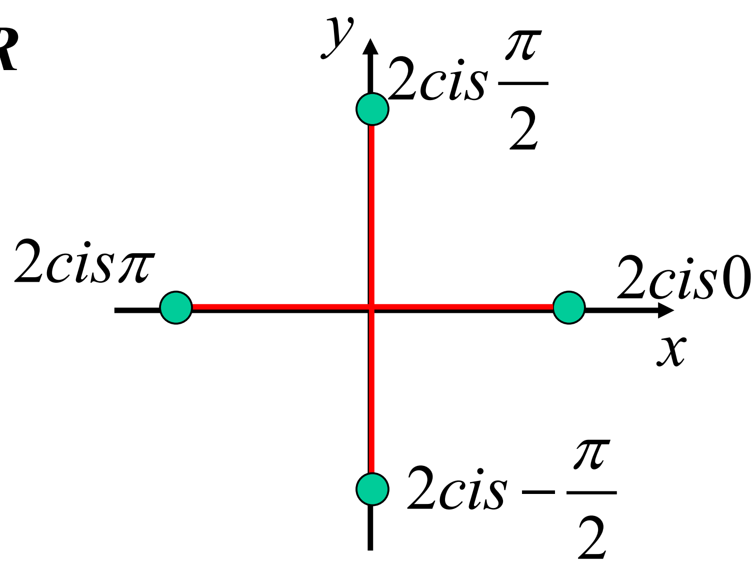
$$x^4 = 16\text{cis } 0$$

$$x = 2\text{cis}\left[\frac{2\pi k}{4}\right] \quad k = 0, \pm 1, 2$$

$$x = 2\text{cis } 0,\ 2\text{cis}\frac{\pi}{2},\ 2\text{cis } -\frac{\pi}{2},\ 2\text{cis}\,\pi$$

$$\underline{x = 2,\ 2i,\ -2i,\ -2}$$

OR



(*iii*) Find the four 4^{th} roots of $1 + \sqrt{3}\, i$ ***OR***

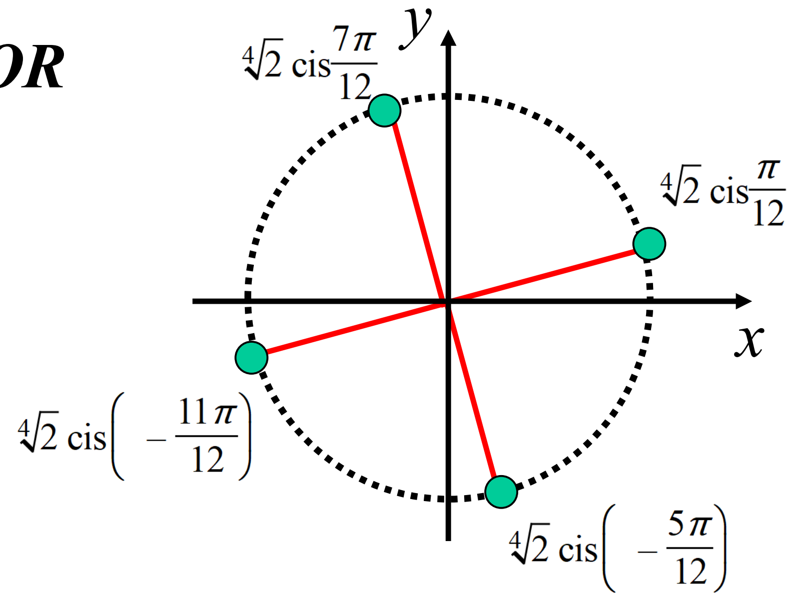
$$z^4 = 1 + \sqrt{3}\, i$$

$$z^4 = 2\text{cis}\frac{\pi}{3}$$

$$z = \sqrt[4]{2}\ \text{cis}\left[\frac{\frac{\pi}{3} + 2\pi k}{4}\right] \quad k = 0, 1, 2, 3$$

$$z = \sqrt[4]{2}\ \text{cis}\frac{\pi}{12}, \sqrt[4]{2}\ \text{cis}\frac{7\pi}{12}, \sqrt[4]{2}\ \text{cis}\frac{13\pi}{12}, \sqrt[4]{2}\ \text{cis}\frac{19\pi}{12}$$

$$z = \sqrt[4]{2}\ \text{cis}\frac{\pi}{12}, \sqrt[4]{2}\ \text{cis}\frac{7\pi}{12}, \sqrt[4]{2}\ \text{cis}\left(-\frac{11\pi}{12}\right), \sqrt[4]{2}\ \text{cis}\left(-\frac{5\pi}{12}\right)$$



If leaving in mod-arg form, use the principal argument

Exercise 3A; 1, 2, 3 abef, 5, 6, 7, 9 to 14, 16 to 18