Solving Quadratics

e.g. $x^2+3x+7=0$

$$x=\frac{-3\pm\sqrt{9-28}}{2}$$

$$=\frac{-3\pm\sqrt{-19}}{2}$$

$$=\frac{-3\pm\sqrt{19}i}{2}$$

$$x=\frac{-3+\sqrt{19}i}{2} \text{ or } x=\frac{-3-\sqrt{19}i}{2}$$

OR

by completing the square

$$x^2+3x+7=0$$

$$\left(x+\frac{3}{2}\right)^2+\frac{19}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{19}{4}i^2=0$$

$$\left(x+\frac{3}{2}+\frac{\sqrt{19}}{2}i\right)\left(x+\frac{3}{2}-\frac{\sqrt{19}}{2}i\right)=0$$

$$x=-\frac{3}{2}+\frac{\sqrt{19}}{2}i \text{ or } x=-\frac{3}{2}-\frac{\sqrt{19}}{2}i$$

Creating Quadratics

All quadratics with real coefficients have conjugate pairs as solutions

$$(x-z)\left(x-\overline{z}\right)=x^2-(z+\overline{z})x+z\overline{z}$$

$$=x^2-2\operatorname{Re}(z)x+z\overline{z}$$

(iii) Find the quadratic equation with roots $4+i$ and 4-i

$$\alpha+\beta=8 \qquad \alpha\beta=17$$

$\therefore$ equation is $x^2-8x+17=0$

Finding Square Roots

$$a + ib = \sqrt{x + iy}$$

$$(a + ib)^2 = x + iy$$

$$a^2 + 2abi - b^2 = x + iy$$

If $\sqrt{x + iy} = a + ib$

then $a^2 - b^2 = x$

$$2ab = y$$

e.g. Find $\sqrt{-12 + 16i}$

$$a^2 - b^2 = -12 \qquad 2ab = 16$$

$$a^2 - \frac{64}{a^2} = -12 \qquad b = \frac{8}{a}$$

$$a^4 + 12a^2 - 64 = 0$$

$$(a^2 - 4)(a^2 + 16) = 0$$

$a^2 = 4$ or $a^2 = -16$

$a = \pm 2$ no real solutions

$$\therefore b = \pm 4$$

$$\sqrt{-12 + 16i} = \pm(2 + 4i)$$

OR

$$a+ib=\sqrt{x+iy}$$

$$a^2-b^2=x$$

$$2ab=y$$

$$\Rightarrow$$

$$x^2+y^2=\left(a^2-b^2\right)^2+4a^2b^2$$

$$=a^4-2a^2b^2+b^4+4a^2b^2$$

$$=a^4+2a^2b^2+b^4$$

If $\sqrt{x+iy}=a+ib$

then $a^2-b^2=x$

$$a^2+b^2=\sqrt{x^2+y^2}$$

Note: $y>0$, then a and b have same sign

$y<0$, then a and b have different sign

e.g. Find $\sqrt{-12+16i}$

$$a^2-b^2=-12$$

$$a^2+b^2=20$$

$$2a^2=8$$

$$a^2=4$$

$$a=2 \qquad \therefore b=4$$

$$\sqrt{-12+16i}=\pm(2+4i)$$

(*ii*) Find $\sqrt{5-4i}$

$$a^2 - b^2 = 5$$

$$a^2 + b^2 = \sqrt{41}$$

$$2a^2 = 5 + \sqrt{41}$$

$$a^2 = \frac{1}{2}\left(5 + \sqrt{41}\right)$$

$$a = \frac{\sqrt{5+\sqrt{41}}}{\sqrt{2}}$$

$$a = \frac{1}{2}\sqrt{10 + 2\sqrt{41}}$$

$$\therefore b^2 = \frac{1}{2}\left(5 + \sqrt{41}\right) - 5$$

$$= \frac{1}{2}\left(-5 + \sqrt{41}\right)$$

$$b = \frac{\sqrt{-5+\sqrt{41}}}{\sqrt{2}}$$

$$b = \frac{1}{2}\sqrt{-10 + 2\sqrt{41}}$$

$$\sqrt{5-4i} = \pm\frac{1}{2}\left(\sqrt{10+2\sqrt{41}} - i\sqrt{-10+2\sqrt{41}}\right)$$

(iii) Solve $z^2+(2+i)z+(2-2i)=0$

$$z=\frac{-(2+i)\pm\sqrt{(2+i)^2-4(2-2i)}}{2}$$

$$=\frac{-(2+i)\pm\sqrt{4+4i-1-8+8i}}{2}$$

$$=\frac{-(2+i)\pm\sqrt{-5+12i}}{2}$$

$$=\frac{-(2+i)\pm(2+3i)}{2}$$

$$=\frac{2i}{2}\quad\text{or}\quad\frac{-4-4i}{2}$$

$$=i\quad\text{or}\quad -2-2i$$

$\sqrt{-5+12i}$

$$a^2-b^2=-5$$

$$a^2+b^2=13$$

$$2a^2 \qquad =8$$

$$a^2=4$$

$$a=2 \qquad \therefore b=3$$

Exercise 1B; 1 to 4 ace etc, 5, 6, 7 cf, 9, 10, 11b, 12, 13, 15ace, 16 ac