Representing Real Numbers

All real numbers can be placed on the number line and described;

- geometrically *(using a picture of the number line)*
- algebraically *(using an inequation or equation)*
- using interval notation *(often used when describing domain & range)*
- using set notation *(formal way of describing all possible numbers)*

Types of Intervals

(i) bounded: interval has two endpoints

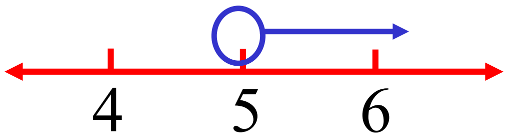
(ii) unbounded: interval has one endpoint

(iii) closed: all endpoints are included

(iv) open: an endpoint is not included

(v) degenerate: a single point

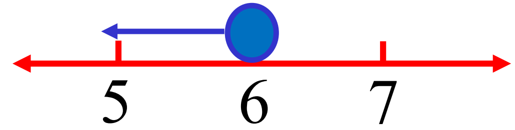
e.g. $(i)\ x > 5$



open unbounded interval

$$(5,\infty) = \{x : x > 5\}$$

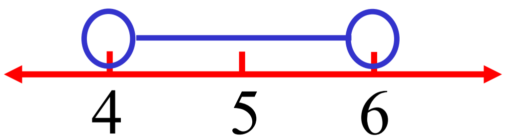
$(ii)\ x \leq 6$



closed unbounded interval

$$(-\infty,6] = \{x : x \leq 6\}$$

$(iii)\ 4 < x < 6$



open bounded interval

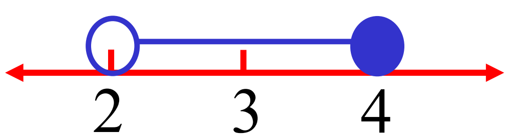
$$(4,6) = \{x : 4 < x < 6\}$$

$(iv)\ -2 \leq x \leq 1$

closed bounded interval

$$[-2,1] = \{x : -2 \leq x \leq 1\}$$

$(v)\ 2 < x \leq 4$



bounded interval

$$(2,4] = \{x : 2 < x \leq 4\}$$

Rational Numbers

Rational numbers can be expressed in the form $\frac{a}{b}$ where a and b are integers.

Irrational Numbers

Irrational numbers are numbers which are not rational.

All irrational numbers can be expressed as a unique infinite decimal.

e.g. Prove $\sqrt{2}$ is irrational

"Proof by contradiction"

Assume $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \frac{a}{b}$ where a and b are integers with no common factors

$b\sqrt{2} = a$

$2b^2 = a^2$

Thus a^2 must be divisible by 2

As prime factors of squares must appear in pairs, any square that is divisible by 2 is also divisible by 4

Thus a^2 must be divisible by 4

$\therefore\ 2b^2 = 4k$ where k is an integer

$b^2 = 2k$

So a^2 and b^2 are both divisible by 2 and must have a common factor

However, a and b have no common factors

so $\sqrt{2}$ is not rational

$\therefore \sqrt{2}$ is irrational

Significant Figures

Irrational numbers cannot be calculated exactly, so sometimes an approximation is required.

When approximating we write a number correct to either;

- a certain number of decimal places; ***OR***
- a certain number of significant figures

Rounding off to a given number of significant figures

Start at the first **non-zero** digit and count to the required number and round *(if the answer is ambiguous, scientific notation should be used)*

e.g. Write correct to the given number of significant figures

$(i)\ 0.050703\ (2) = \underline{0.051}$

$(ii)\ 0.050703\ (3) = \underline{0.0507}$

$(iii)\ 0.050703\ (4) = \underline{0.05070}$

$(iv)\ 3000\ (2) = \underline{3000} \quad = \underline{3.0 \times 10^3}$

$(vii)\ 3000\ (3) = \underline{3000} \quad = \underline{3.00 \times 10^3}$

$(vi)\ 3000\ (4) = \underline{3000} \quad = \underline{3.000 \times 10^3}$

Finding the number of significant figures

Start at the first **non-zero** digit and count the number of digits until the end of the number

e.g. (i) 0.050703 $(5 \text{ significant figures})$

(ii) 0.010031 $(5 \text{ significant figures})$

(iii) 0.0100310 $(6 \text{ significant figures})$

(iv) 1200 $(2, 3 \text{ or } 4 \text{ significant figures})$

$(4 \text{ significant figures})$

If the answer is ambiguous use the ***largest*** *answer*

Exercise 2B; 1cdfikl, 3, 6, 7, 8, 11hkl, 12a, 15, 16