

# *Differentiating Logarithms*

$$y = \log f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\log a)f(x)}$$

e.g. (i)  $y = \log(3x+5)$

$$\frac{dy}{dx} = \frac{3}{3x+5}$$

(ii)  $y = \log x^3$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3x^2}{x^3} \\ &= \frac{3}{x}\end{aligned}$$

**OR**

$$y = \log x^3$$

$$y = 3 \log x$$

$$\frac{dy}{dx} = \frac{3}{x}$$

$$(iii) \ y = \log(\log x)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x \log x} \\ &= \frac{1}{x \log x} \end{aligned}$$

$$(iv) \ y = \log(x+3)(x+2)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+3)(1) + (x+2)(1)}{(x+3)(x+2)} \\ &= \frac{2x+5}{(x+3)(x+2)} \end{aligned}$$

**OR**

$$y = \log(x+3) + \log(x+2)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x+3} + \frac{1}{x+2} \\ &= \frac{(x+2) + (x+3)}{(x+3)(x+2)} \\ &= \frac{2x+5}{(x+3)(x+2)} \end{aligned}$$

$$\begin{aligned}
 (v) \quad y &= \log \left\{ \frac{x+5}{x+2} \right\} \\
 \frac{dy}{dx} &= \frac{(x+2)(1) - (x+5)(1)}{(x+2)^2} \\
 &= \frac{-3}{(x+2)^2} \times \frac{(x+2)}{(x+5)} \\
 &= \frac{-3}{(x+2)(x+5)}
 \end{aligned}$$

**OR**

$$\begin{aligned}
 y &= \log(x+5) - \log(x+2) \\
 \frac{dy}{dx} &= \frac{1}{x+5} - \frac{1}{x+2} \\
 &= \frac{(x+2) - (x+5)}{(x+5)(x+2)} \\
 &= \frac{-3}{(x+2)(x+5)}
 \end{aligned}$$

$$\begin{aligned}
 (vi) \quad y &= \log_{10} 6x \\
 \frac{dy}{dx} &= \frac{6}{(\log 10)6x} \\
 &= \frac{1}{x \log 10}
 \end{aligned}$$

**Exercise 6G; 1aei, 2cf, 4e, 5e, 6a, 8b, 9, 10acg, 11af, 12abdfi, 14ag, 15b, 16, 17**

**Exercise 6H; 3b, 4, 6, 7, 9, 10, 12, 14, 15, 17**

**Exercise 6K; 2a, 3b, 9ab, 11, 17b, 20**