Conditional Probability

The conditional probability of an event *A*, given that event *B* has already occurred is given by;

 $P(A | B) = \frac{P(A \cap B)}{P(B)} \quad \text{or} \quad P(A | B) = \frac{|A \cap B|}{|B|}$ Note: if P(A | B) = P(A) then A and B are independent events or $P(A \cap B) = P(A) \times P(B)$

e.g. (*i*) In a mixed language class, students study French, Latin and Japanese. The number of students who study each language are shown in the Venn diagram.



What is the probability that a student who studies Japanese also studies French?

$$P(\text{French} | \text{Japanese}) = \frac{3}{10}$$

(ii) Two boxes each contain four stones that differ only in colour.

Box 1 contains four black stones

Box 2 contains two black stones and two white stones

- A box is chosen at random and one stone is randomly drawn from it
- a) What is the probability that the randomly drawn stone is black

$$P(\text{black}) = \frac{1}{2} + \frac{1}{2} \times \frac{2}{4}$$
 OR $P(\text{black}) = \frac{6}{8}$
 $= \frac{3}{4}$ $= \frac{3}{4}$

b) It is not known from which box the stone has been drawn.

Given that the stone drawn is black, what is the probability that it was drawn from Box 1?

$$P(\text{Box 1}|\text{black}) = \frac{P(\text{black and Box 1})}{P(\text{black})}$$
$$= \frac{\frac{1}{2} \times 1}{\frac{3}{4}}$$
$$= \frac{2}{3}$$

(*iii*) In a particular school 55% are male and 45% are female. Of the male students 13% say Monday is their favourite day, while 18% of the females say Monday is their favourite.

Find the probability that a student chosen at random is a male whose favourite day is Monday.

 $P(\text{Male}) = 0.55 \qquad P(\text{Monday}|\text{Male}) = 0.13$ $P(\text{Monday}|\text{Male}) = \frac{P(\text{Monday & Male})}{P(\text{Male})}$ $0.13 = \frac{P(\text{Monday & Male})}{0.55}$ $P(\text{Monday & Male}) = 0.13 \times 0.55$ = 0.0715

Exercise 12G; 1ac, 2, 3, 5, 6ac, 7ac, 8ace, 10, 11, 13, 14, 15, 17, 18, 19, 21, 22, 25