Completing the Square

e.g. $(i)\ x^2+6x-7=0$

$x^2+6x=7$ ← move the constant

$x^2+6x+3^2=7+3^2$ ← add half the coefficient of 'x' squared

$x^2+6x+9=16$

$(x+3)^2=16$ ← factorise to a perfect square

$x+3=\pm 4$

$x=-3\pm 4$

$x=-7 \quad \text{or} \quad x=1$

$$(ii)\ ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$(iii)\ x^2 - 6x + 6 = 0$$

$$(x-3)^2 - 3 = 0$$

$$\left(x - 3 + \sqrt{3}\right)\left(x - 3 - \sqrt{3}\right) = 0$$

$$x = 3 - \sqrt{3} \quad \text{or} \quad x = 3 + \sqrt{3}$$

Exercise 1H; 1adh, 2ch, 3adg, 4bfi, 5be, 6bc, 7, 8