Trig Integrals I

(1) Basic Integrals

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$$= -\log|\cos x| + c \quad \textbf{\textit{OR}} \quad \log|\sec x| + c$$

$$\int \sec x \tan x\,dx = \sec x + c$$

(2) Complementary Ratios

$$\int \text{complementary trig ratio} = -\text{complement of the answer}$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$$

$$\int \cot x\,dx = \log|\sin x| + c$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$$

$$\int \tan x\,dx = \log \sec x$$

$$\int \cot x\,dx = -\log \operatorname{cosec} x$$

cot is the complement of tan

so the answer is minus and the complement of sec is cosec

(3) Squares of Trig Functions

$$\int \sin^2 x\,dx = \frac{1}{2}\int (1 - \cos 2x)\,dx \qquad = \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

$$\left(= \frac{x}{2} - \frac{1}{2}\sin x\cos x + c \right)$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int (1 + \cos 2x)\,dx \qquad = \frac{x}{2} + \frac{1}{4}\sin 2x + c$$

$$\left(= \frac{x}{2} + \frac{1}{2}\cos x\sin x + c \right)$$

$$\int \sec^2 x\,dx = \tan x + c \qquad \int \operatorname{cosec}^2 x\,dx = -\cot x + c$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx \qquad = \tan x - x + c$$

$$\int \cot^2 x\,dx = \int (\operatorname{cosec}^2 x - 1)\,dx \qquad = -\cot x - x + c$$

e.g. (*i*) $\int \cos^2 3x\, dx = \frac{1}{2}\int (1 + \cos 6x)dx$

$$= \frac{x}{2} + \frac{1}{12}\sin 6x + c$$

(*ii*) $\int \sin x \cos 2x\, dx$

$$= \frac{1}{2}\int (\sin 3x + \sin(-x))dx$$

$$= \frac{1}{2}\int (\sin 3x - \sin x)dx$$

$$= -\frac{\cos 3x}{6} + \frac{\cos x}{2} + c$$

(*iii*) $\int_0^{\frac{\pi}{4}} \sin x \cos^3 x dx$

$$= -\int_0^{\frac{\pi}{4}} -\sin x \cos^3 x dx$$

$$= \left[\frac{\cos^4 x}{4}\right]_{\frac{\pi}{4}}^{0}$$

$$= \frac{1}{4}\left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{16}$$

Exercise 12C; 4c, 5ace, 6, 8bdfh, 9, 10ab i, 11ad, 12, 13, 14