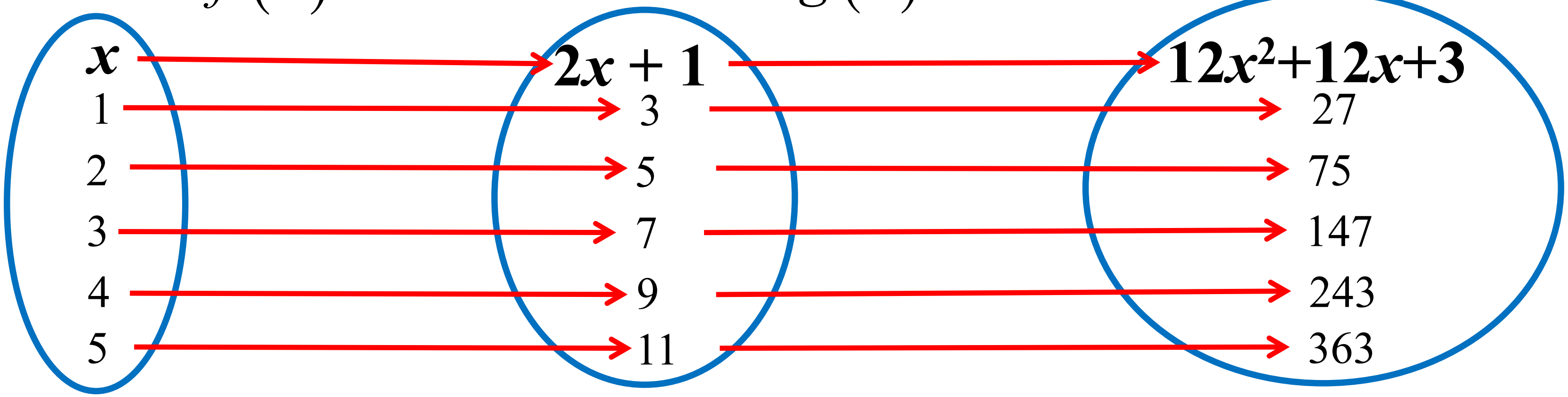
Composite Functions

A **composite function** is when two or more functions combine to create a new function.

$$f(x) = 2x + 1 \qquad g(x) = 3x^2$$



$$g(f(x)) = 12x^2 + 12x + 3$$

Note: In general $f(g(x))$ and $g(f(x))$ are different functions

$$f(g(x)) = f(3x^2)$$

$$= 2(3x^2) + 1$$

$$= 6x^2 + 1$$

Domain and Range of composite functions

For a value of x to be in the domain of the composite function $f(g(x))$, two conditions must be met;

1) x must be in the domain of $g(x)$

2) $g(x)$ must be in the domain of $f(x)$

A value of $f(g(x))$ is in the range of the composite function only if x is in the domain of $f(g(x))$

e.g. $f(x)=\dfrac{2x}{4-x}$ and $g(x)=\dfrac{1}{x^2}$

(i) Find the domain and range of $f(g(x))$

$$g(x)=\frac{1}{x^2}$$

domain: $x \in \mathbb{R}\backslash 0$

range: $g(x) \in \mathbb{R} : g(x) > 0$

$$f(x)=\frac{2x}{4-x}$$

domain: $x \in \mathbb{R}\backslash 4$

range: $f(x) \in \mathbb{R}\backslash -2$

x must be in the domain of $g(x)$

$$x \neq 0$$

$g(x)$ must be in the domain of $f(x)$

$$g(x) \neq 4 \rightarrow \frac{1}{x^2} \neq 4$$

$$x \neq \pm\frac{1}{2}$$

$$\text{domain: all real } x \text{ except } x = 0, \pm\frac{1}{2}$$

A value of $f(g(x))$ is in the range of the composite function only if x is in the domain of $f(g(x))$

$$f(g(x)) = \frac{2\left(\frac{1}{x^2}\right)}{4 - \frac{1}{x^2}}$$

$$= \frac{2}{4x^2 - 1}$$

$$\text{range } f(g(x)) : f(g(x)) \leq -2 \cup f(g(x)) > 0$$

$$\text{but } x \neq 0 \rightarrow f(g(x)) \neq \frac{2}{4(0)^2 - 1} = -2$$

$$\text{range } f(g(x)) : f(g(x)) < -2 \cup f(g(x)) > 0$$

(ii) Find the domain and range of $g\left(f\left(x\right)\right)$

x must be in the domain of $f(x)$

$$x \neq 4$$

$f(x)$ must be in the domain of $g(x)$

$$f\left(x\right) \neq 0 \rightarrow \frac{2x}{4-x} \neq 0$$

domain: all real x except $x = 0, 4$

A value of $g\left(f\left(x\right)\right)$ is in the range of the composite function only if x is in the domain of $g\left(f\left(x\right)\right)$

$$g\left(f\left(x\right)\right) = \frac{1}{\left(\frac{2x}{4-x}\right)^2}$$

$$= \frac{\left(4-x\right)^2}{4x^2}$$

range $g\left(f\left(x\right)\right) : g\left(f\left(x\right)\right) \geq 0$

but $x \neq 4 \rightarrow g\left(f\left(x\right)\right) \neq \frac{\left(4-4\right)^2}{4\left(4\right)^2} = 0$

range $g\left(f\left(x\right)\right) : g\left(f\left(x\right)\right) > 0$

Exercise 4E;1bc, 3, 5, 7, 8, 9, 10, 12, 13, 14, 16, 17

If the domain is the empty set, then the function is called the **empty function**