Integration By Parts

When an integral is a product of two functions and neither is the derivative of the other, we integrate by parts.

$$\int u\,dv = uv - \int v\,du$$

Proof:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$uv = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx$$

$$uv = \int u\,dv + \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

u should be chosen so that differentiation makes it a simpler function.

dv should be chosen so that it can be integrated e.g. $(i)\ \int x\sin x\,dx$ $\qquad u = x \qquad v = -\cos x$

$\qquad du = dx \qquad dv = \sin x\,dx$

$$= -x\cos x + \int \cos x\,dx$$

$$= \sin x - x\cos x + c$$

$(ii)\int \log x\,dx$ $\qquad u = \log x \qquad v = x$

$\qquad du = \frac{dx}{x} \qquad dv = dx$

$$= x\log x - \int dx$$

$$= x\log x - x + c$$

$$(iii)\int_0^1 xe^{-7x}dx \qquad u = x \qquad v = -\frac{1}{7}e^{-7x}$$

$$du = dx \qquad dv = e^{-7x}dx$$

$$= \left[-\frac{1}{7}xe^{-7x}\right]_0^1 + \frac{1}{7}\int_0^1 e^{-7x}dx$$

$$= \left[-\frac{1}{7}xe^{-7x} - \frac{1}{49}e^{-7x}\right]_0^1$$

$$= \left\{-\frac{1}{7}e^{-7} - \frac{1}{49}e^{-7}\right\} - \left\{0 - \frac{1}{49}\right\}$$

$$= -\frac{8}{49}e^{-7} + \frac{1}{49}$$

$$(iv)\int e^x \cos x dx$$

$u = e^x \qquad v = \sin x$

$du = e^x dx \qquad dv = \cos x dx$

$$= e^x \sin x - \int e^x \sin x dx$$

$u = e^x \qquad v = -\cos x$

$du = e^x dx \qquad dv = \sin x dx$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x dx$$

$$\therefore 2\int e^x \cos x dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x dx = \frac{1}{2}e^x \sin x + \frac{1}{2}e^x \cos x + c$$

Exercise 4F; 1abdf, 2cef, 3c, 4c, 5bc, 6bc, 7ac, 8b, 9a, 10ac, 11bc, 12, 13acd, 14ac, 15a, 16b, 17