Projectile Motion

$$\ddot{x} = 0 \qquad \ddot{y} = -g$$

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta - gt$$

$$x = vt\cos\theta \qquad y = vt\sin\theta - \frac{1}{2}gt^2$$

Note: parametric coordinates of a parabola

$$t = \frac{x}{v\cos\theta}$$

$$y = -\frac{gx^2}{2v^2\cos^2\theta} + \frac{x\sin\theta}{\cos\theta}$$

$$y = -\frac{gx^2}{2v^2}\sec^2\theta + x\tan\theta$$

$$y = -\frac{gx^2}{2v^2}\left(\tan^2\theta + 1\right) + x\tan\theta$$

Common Questions

(1) What is the range of the particle? ***roots of the quadratic***

$$-\frac{gx^2}{2v^2}\sec^2\theta + x\tan\theta = 0$$

$$\frac{x\sin\theta}{\cos\theta} - \frac{gx^2}{2v^2\cos^2\theta} = 0$$

$$\frac{x}{\cos\theta}\left(\sin\theta - \frac{gx}{2v^2\cos\theta}\right) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{2v^2\sin\theta\cos\theta}{g}$$

$$x = \frac{v^2}{g}\sin 2\theta$$

$\therefore$ range of the particle is $\frac{v^2}{g}\sin 2\theta$ metres

(2) What is the greatest height of the particle? ***vertex of the parabola***

$$\text{greatest height} = -\frac{\Delta}{4a}$$

$$= -\frac{\tan^2\theta}{4\left(\frac{-g\sec^2\theta}{2v^2}\right)}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{v^2\cos^2\theta}{2g}$$

$$= \frac{v^2\sin^2\theta}{2g}$$

$\therefore$ greatest height is $\frac{v^2\sin^2\theta}{2g}$ metres

(4) What angle does the particle make with the ground?

(i) find slope of the tangent

(ii) m=tanθ

e.g. A ball is thrown with an initial velocity of 25 m/s at an angle of $\theta = \tan^{-1}\frac{3}{4}$ to the ground. Determine;

a) the cartesian equation of the path

$$x = 20t \qquad y = -5t^2 + 15t$$

$$t = \frac{x}{20} \qquad y = -5\left(\frac{x}{20}\right)^2 + 15\left(\frac{x}{20}\right)$$

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$

b) greatest height is y value of the vertex $\quad y = -\dfrac{\Delta}{4a}$

$$= -\frac{9}{16} \times -\frac{20}{1}$$

$$= \frac{45}{4}$$

$\therefore$ greatest height is $11\frac{1}{4}$ m above the ground

c) range

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$

$$= \frac{x}{4}\left(3 - \frac{x}{20}\right)$$

roots are 0 and 60

$\therefore$ range is 60m

d) velocity and direction of the ball after $\frac{1}{2}$ second

$$y = \frac{-x^2}{80} + \frac{3x}{4} \qquad \text{when } t = \frac{1}{2}, x = 10 \qquad \frac{dy}{dx} = \frac{-10}{40} + \frac{3}{4}$$

$$\frac{dy}{dx} = \frac{-x}{40} + \frac{3}{4} \qquad = \frac{1}{2}$$

$$\tan\theta = \frac{1}{2}$$

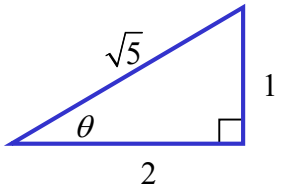
$$\theta = 26^\circ 34'$$

Exercise 10B; 2, 3, 4, 5, 8, 9, 10, 11, 12

$$v = \frac{x}{t\cos\theta}$$

$$= \frac{10}{\frac{1}{2} \times \frac{2}{\sqrt{5}}}$$

$$= 10\sqrt{5}$$



$\therefore$ after $\frac{1}{2}$ second, velocity $= 10\sqrt{5}$ m/s and it is traveling at an angle of $26^\circ 34'$ to the horizontal