Exponential Function

Differentiating Exponentials

$$f(x) = a^x \qquad\qquad f(x+h) = a^{x+h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h \to 0} \frac{a^x(a^h - 1)}{h}$$

$$= a^x \lim_{h \to 0} \frac{a^h - 1}{h}$$

Let's look at this limit in a bit more detail

$$\lim_{h \to 0} \frac{a^h - 1}{h} = \lim_{h \to 0} \frac{a^{0+h} - a^0}{h}$$

$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= f'(0)$$

$= $ slope of tangent at $x = 0$

If $f(x) = a^x$

then $f'(x) = ma^x$

where m = slope of tangent at $x = 0$

Euler's Number (e)

e is an **irrational number**, it is defined as;

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n$$

$$e \approx 2.718281828\ldots$$

The exponential function, base e, has a slope of 1 at $x = 0$

$$\text{If } f(x) = e^x$$

$$f'(0) = 1$$

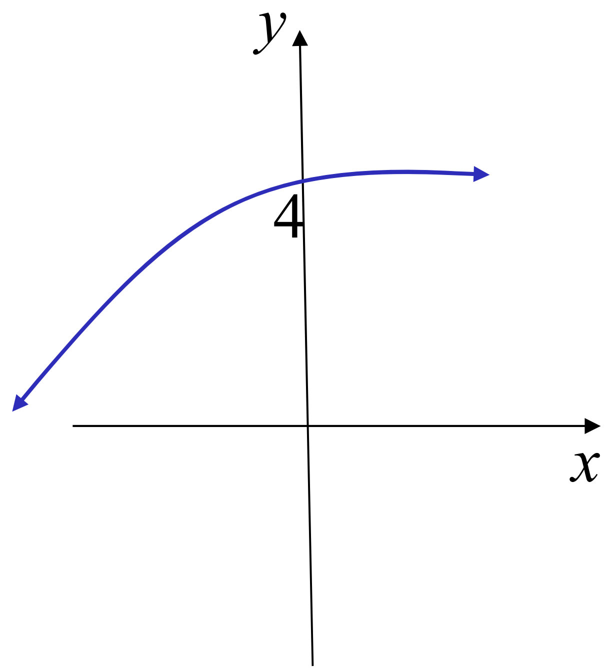
$$\text{thus } f'(x) = e^x$$

so the slope of the basic exponential function, base e, is equal to the height of the curve above the x-axis i.e. $f'(x) = y$

e.g. (i) Find $e^3 = 20.086$ (to 3 dp)

(ii) Sketch $y = 5 - e^{-x}$

1. *basic curve:* $y = e^x$
2. *reflect in y-axis*
3. *reflect in x-axis*
4. *shift up 5 units*



Exercise 11A; 1, 2

Exercise 11B; 1aegh, 2ac, 3cf, 4c, 6b ii, 7ab, 9cf, 10ab