Mechanics

Newton's Laws Of Motion

Law 1

" Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter, nisi quatenus a viribus impressis cogitur statum illum mutare."

Everybody continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

Law 2

" Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum rectam qua vis illa impritmitur."

The change of motion is proportional to the motive force impressed, and it is made in the direction of the right line in which that force is impressed.

$$\text{Motive force } \alpha \text{ change in motion}$$

$$F \alpha \text{ acceleration}$$

$$F = \text{constant} \times \text{acceleration}$$

$$\therefore F = ma \qquad \text{(constant = mass)}$$

Law 3

"Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actions in se mutuo semper esse aequales et in partes contrarias dirigi."

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed in contrary parts.

Vectors & Forces

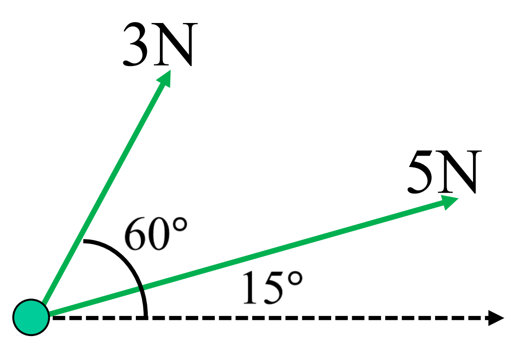
Forces are Vectors

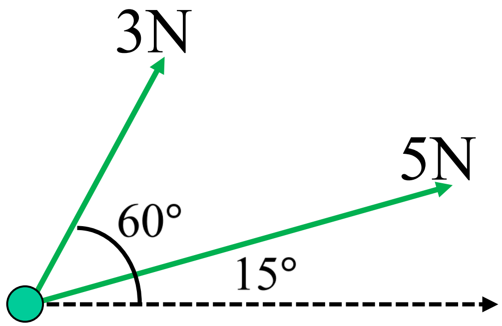
Any force that is not vertical or horizontal must be resolved into vertical and horizontal components. The original force will be the hypotenuse of a right angled triangle.

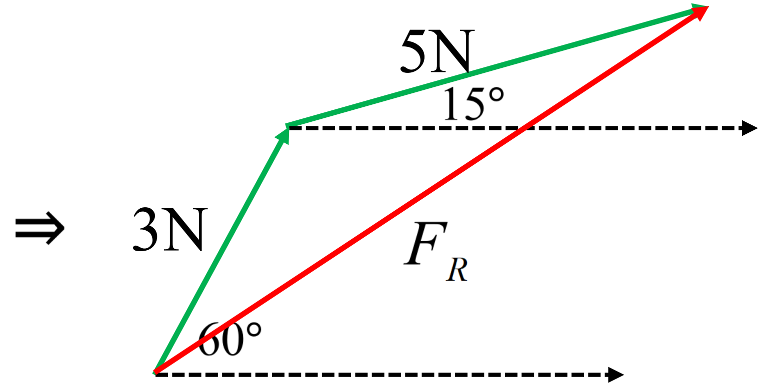
Resultant Vectors

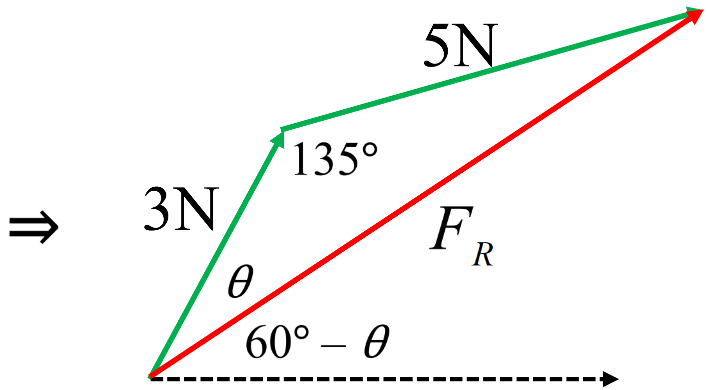
The **resultant vector** is the sum of all the **component vectors** acting at a point.

e.g. Find the magnitude and direction of the resultant force of the forces 3 N and 5N acting on a particle as shown in this diagram









$$\frac{\sin\theta}{5} = \frac{\sin 135°}{F_R}$$

$$\sin\theta = \frac{5\sin 135°}{F_R}$$

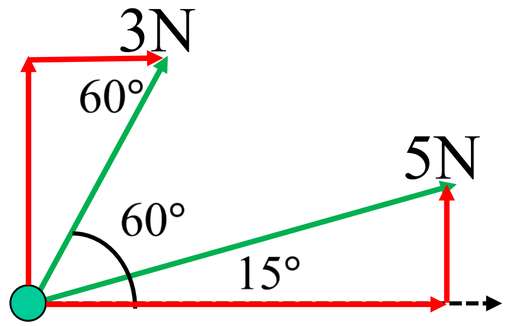
$$\theta = 28.41....°$$

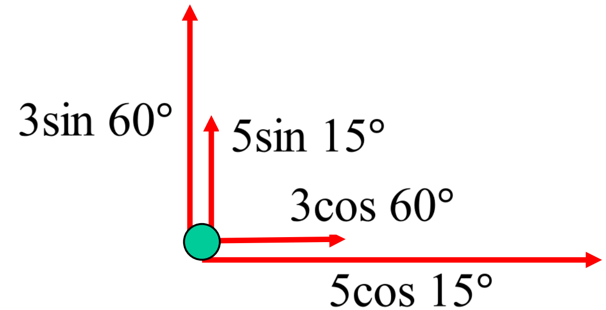
$$\text{direction} = (60 - 28.41...)°$$
$$= 32.59...°$$

$$|F_R|^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos 135°$$

$$|F_R| = 7.43...$$

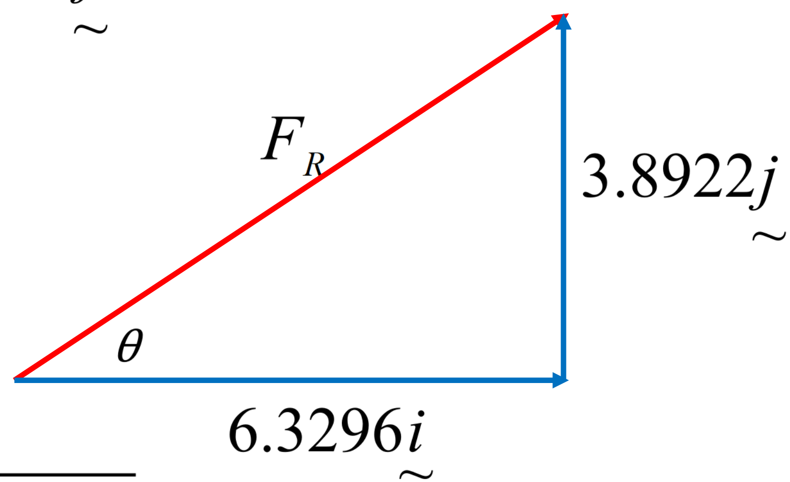
or by projecting vectors into vertical and horizontal components





$$F_R = (3\cos 60° + 5\cos 15°)\underset{\sim}{i} + (3\sin 60° + 5\sin 15°)\underset{\sim}{j}$$

$$= 6.3296\underset{\sim}{i} + 3.8922\underset{\sim}{j}$$



$$|F_R| = \sqrt{6.3296^2 + 3.8922^2}$$

$$|F_R| = 7.43...$$

$$\tan\theta = \frac{3.8922}{6.3296}$$

$$\theta = 32.59...°$$

Types of Forces

(i) Resistive Forces

Whenever a body moves in a medium (air, water, etc) it is subjected to a resistance force that acts in the opposite direction to the motion.

The resistance is proportional to a power of the velocity (usually v or v^2) i.e. the faster you go, the greater the resistance.

$$F_{\text{resistance}} = kv \quad \text{or} \quad kv^2$$

(ii) Gravitational Forces (Weight)

The force of attraction on the object from the earth, called the **force of gravity.** For a particular object it is called the **weight** of the object.

$$F_{\text{gravity}} = mg$$

Forces in Pairs

Newton's third law

If an object A exerts a force on an object B, then B exerts a force on A of the same magnitude in the opposite direction

(iii) Normal Contact Forces

When an object is in contact with a surface, there is a force on the object at right angles to the surface in contact. This is called the **normal contact force** or simply the ***normal force***

(iv) Frictional Forces

When an object slides over a fixed surface, the **frictional force** has its limiting value (a magnitude the force cannot exceed) and acts in a direction opposite to the direction of motion.

$$F_{\text{limiting friction}} = \mu N$$

$$= \text{coefficient of friction} \times \text{normal contact force}$$

(v) Tension Forces

A light string exerts forces of equal magnitude on the objects attached to its two ends.

These forces act along the line of the string, and are directed inward (i.e. away from the objects) at each end.

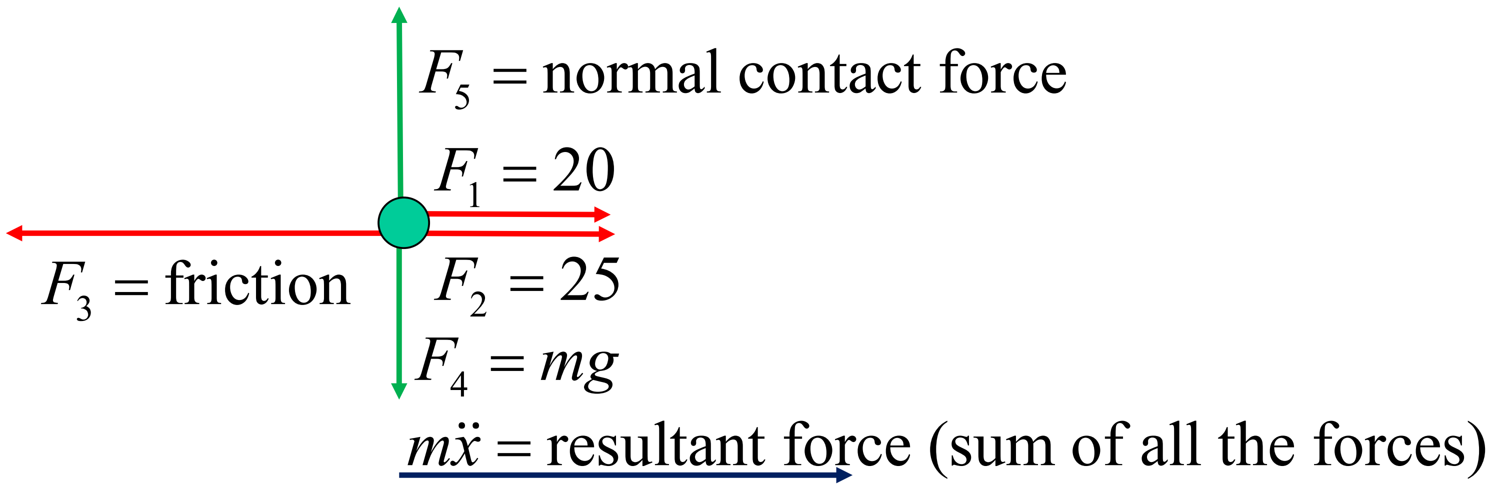
The magnitude of the force at either end is called the **tension** in the string.

In most problems it is usually more convenient to treat the direction of motion as positive and the resistance as negative.

Always use a diagram to show all forces acting on the object

e.g. (i) A heavy box of mass 32 kg has a handle on one side. Two people try to move it across the floor. One pulls horizontally on the handle with a force of 20 N, the other pushes from the other side of the box with a force of 25 N, but the box does not move.

a) Draw a diagram of all the forces acting on the box.



b) Find the frictional force.

$$m\ddot{x} = F_1 + F_2 - F_3$$

horizontal forces = 0

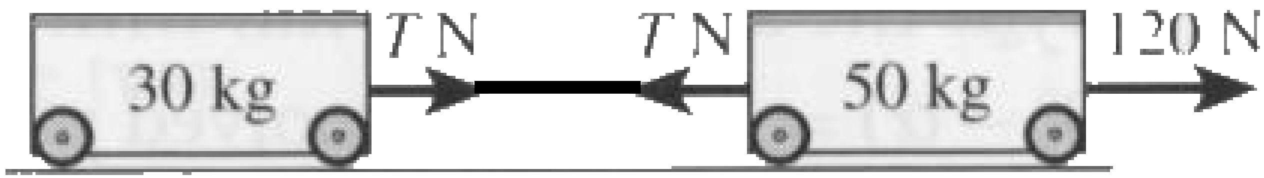
$$0 = 20 + 25 - F_3$$

$$F_3 = 45$$

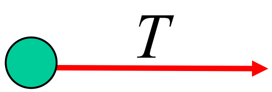
Frictional force is 45 N

(ii) Suppose that the truck on the left has mass 30 kg, and the truck on the right has mass 50 kg. The truck on the right is pulled along with a force of 120 N. Neglecting friction, calculate the tension in the chain.

Note: only horizontal forces are shown in the diagram



Forces on left truck

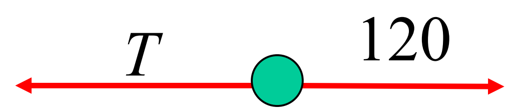


$$m\ddot{x} = T$$

$$30a = T$$

$$a = \frac{T}{30}$$

Forces on right truck



$$m\ddot{y} = 120 - T$$

$$50a = 120 - T$$

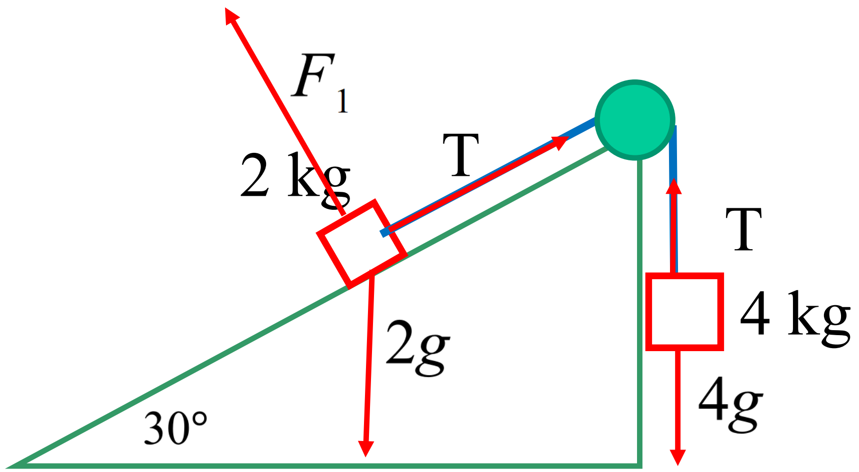
$$\frac{5T}{3} = 120 - T$$

$$\frac{8T}{3} = 120$$

$$\underline{T = 45}$$

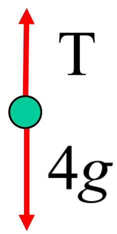
(iii) A mass of 2 kg, resting on a smooth plane inclined at 30º to the horizontal, is connected to a mass of 4 kg by a light elastic string which passes over a small pulley.

a) Draw a force diagram of the problem



b) Find the tension in the string, if the 4 kg block is moving down

Vertical forces on 4 kg mass



$$F_{R4} = 4g - T$$

$$4a = 4g - T$$

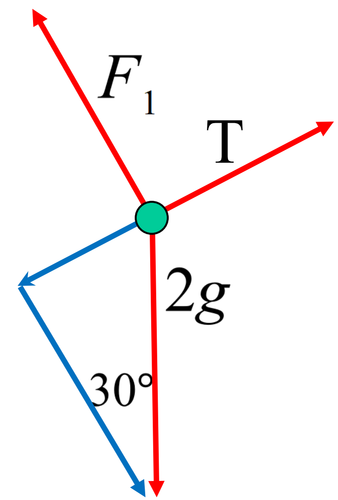
$$4g - T = 2T - 4g\sin 30°$$

$$3T = 4g + 4g\sin 30°$$

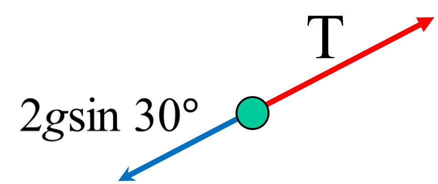
$$T = \frac{4g(1 + \sin 30°)}{3}$$

$$T = 19.6 \text{ N}$$

Forces on 2 kg mass



Forces up the plane on 2 kg mass



$$F_{R_2} = T - 2g\sin 30°$$

$$2a = T - 2g\sin 30°$$