Exponential Equations

e.g. $(i)\ 5^x = 125$

$$x = 3$$

$(ii)\ 4^{2x+1} = \frac{1}{2\sqrt{2}}$

$$\left(2^2\right)^{2x+1} = 2^{-\frac{3}{2}}$$

$$4x + 2 = -\frac{3}{2}$$

$$4x = -\frac{7}{2}$$

$$x = -\frac{7}{8}$$

$(iii)\ 7^x = 32$

$$x = \log_7 32$$

$$= \frac{\log 32}{\log 7}$$

$$= 1.781 \text{ (to 3 dp)}$$

Change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$(iv)\ 3^x = 5^{x-2}$$

$$\log 3^x = \log 5^{x-2}$$

$$x\log 3 = (x-2)\log 5$$

$$x(\log 3 - \log 5) = -2\log 5$$

$$x = \frac{2\log 5}{\log 5 - \log 3}$$

$$= 6.301$$

$$(v)\ 9^x - 4(3^x) + 3 = 0$$

$$\text{let } m = 3^x$$

$$m^2 = (3^x)^2 = 3^{2x} = (3^2)^x = 9^x$$

$$m^2 - 4m + 3 = 0$$

$$(m-3)(m-1) = 0$$

$$m = 3 \quad \text{or} \quad m = 1$$

$$3^x = 3 \quad \text{or} \quad 3^x = 1$$

$$\therefore x = 1 \quad \text{or} \quad x = 0$$

Inequations & Continually Increasing or Decreasing Functions

When a function is applied to an inequation, the inequality sign;

* is maintained if the function is **continually increasing**

* is reversed if the function is **continually decreasing**

e.g. $(i)\ 1 < 2^x < 32$

$$\log_2 1 < \log_2 2^x < \log_2 32$$

$$\underline{0 < x < 5}$$

As both the exponential and logarithmic functions are **continually increasing**, inequalities are preserved.

$(ii)\ \log_3 x > 2$

$$3^{\log_3 x} > 3^2$$

$$\underline{x > 9}$$

Exercise 8E; 2eim, 3bhn, 4dg, 5c (i, iv), 6g, 7f, 11a, 12bce, 14a, 15ac