

# *Inverse Trigonometric Functions*

$y = \sin^{-1} x$

$y = \sin x$

Domain: all real  $x$

Range:  $-1 \leq y \leq 1$

***NO INVERSE***

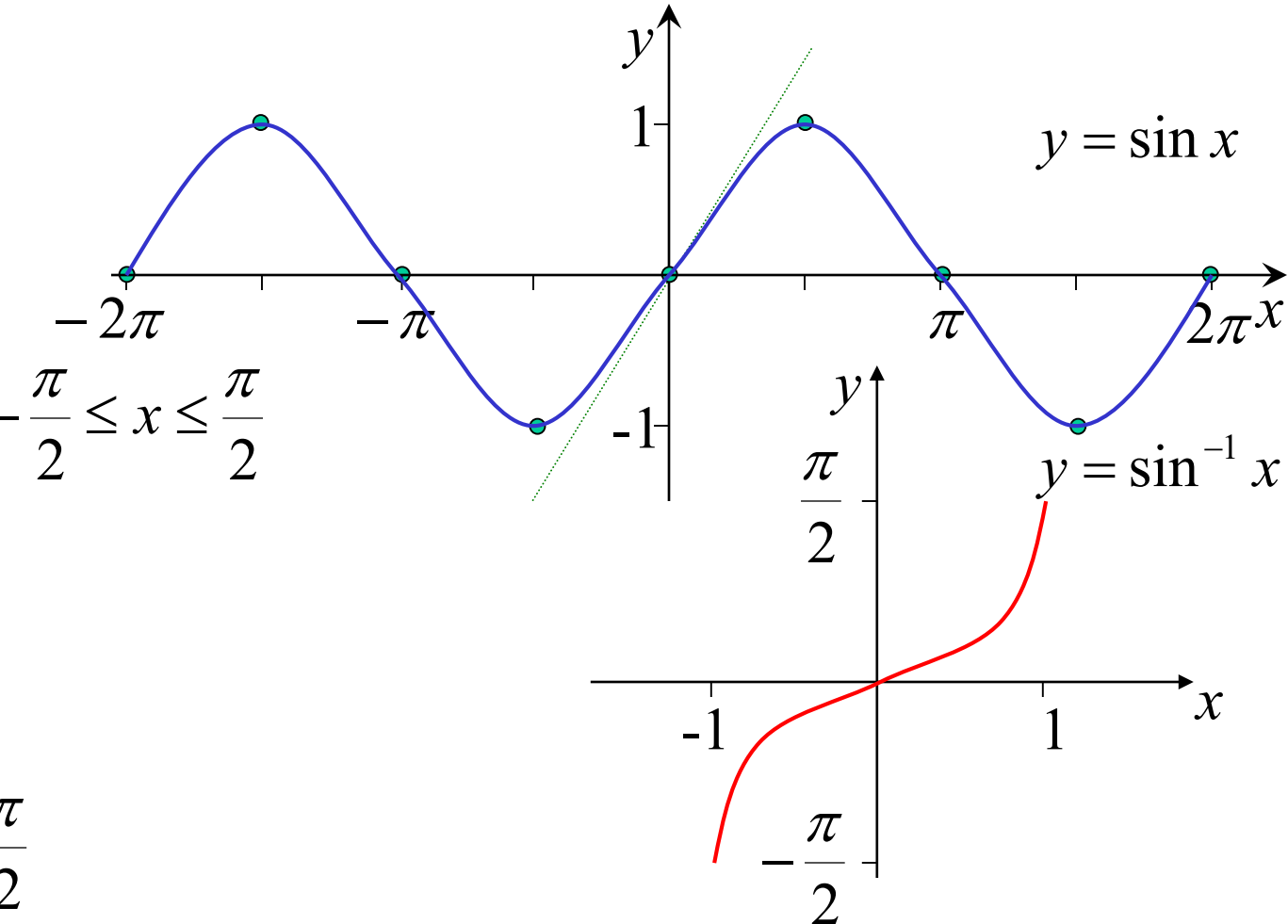
Restricted Domain:  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

Range:  $-1 \leq y \leq 1$

$f^{-1} : y = \sin^{-1} x$

Domain:  $-1 \leq x \leq 1$

Range:  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

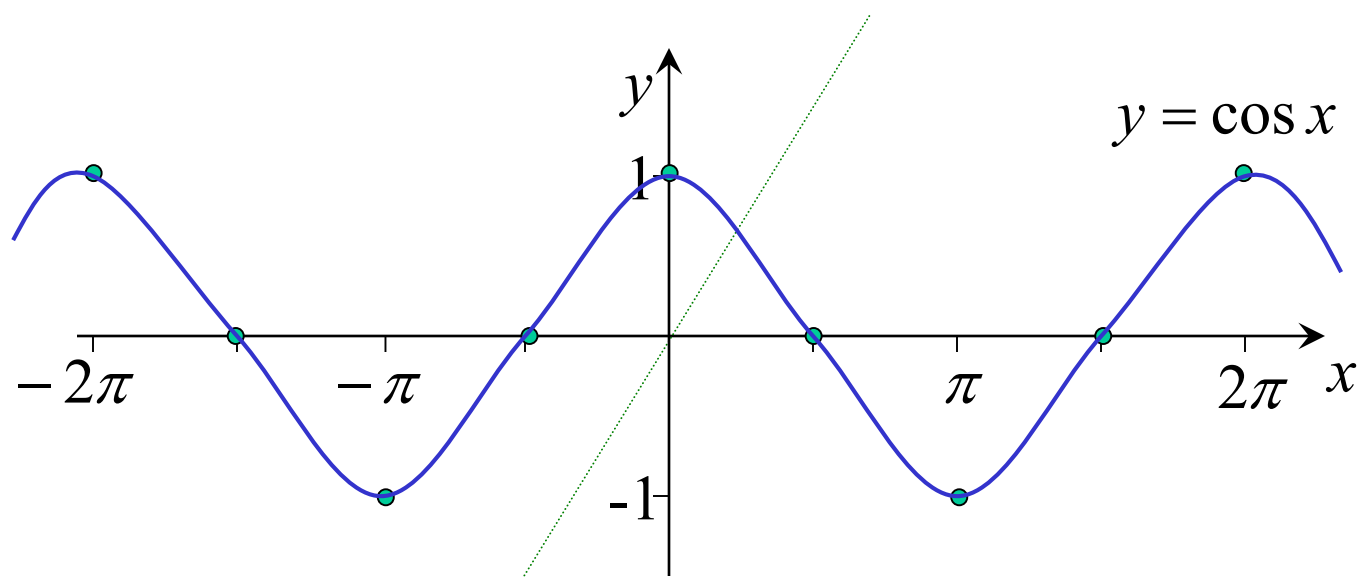


$y = \cos^{-1} x$

$y = \cos x$

Domain: all real  $x$

Range:  $-1 \leq y \leq 1$



***NO INVERSE***

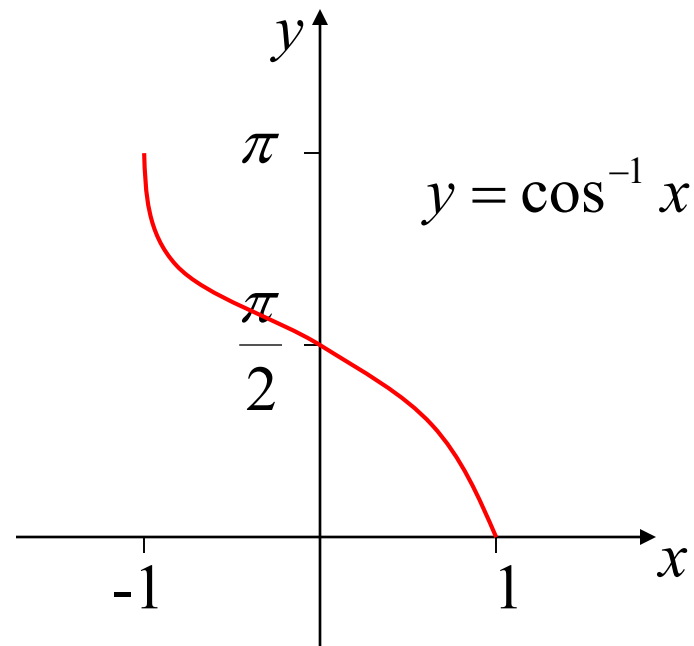
Restricted Domain:  $0 \leq x \leq \pi$

Range:  $-1 \leq y \leq 1$

$f^{-1} : y = \cos^{-1} x$

Domain:  $-1 \leq x \leq 1$

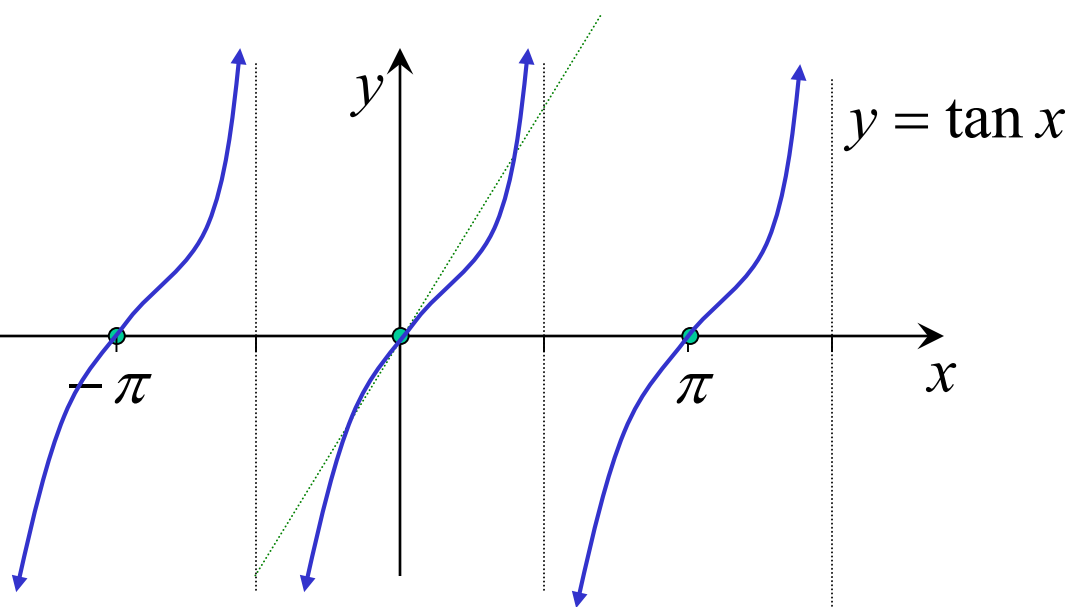
Range:  $0 \leq y \leq \pi$



$y = \tan^{-1} x$

$y = \tan x$

Domain: all real  $x$  except  
 $x = \frac{\pi}{2} + \pi k$ , where  
 $k$  is an integer



Range: all real  $y$

***NO INVERSE***

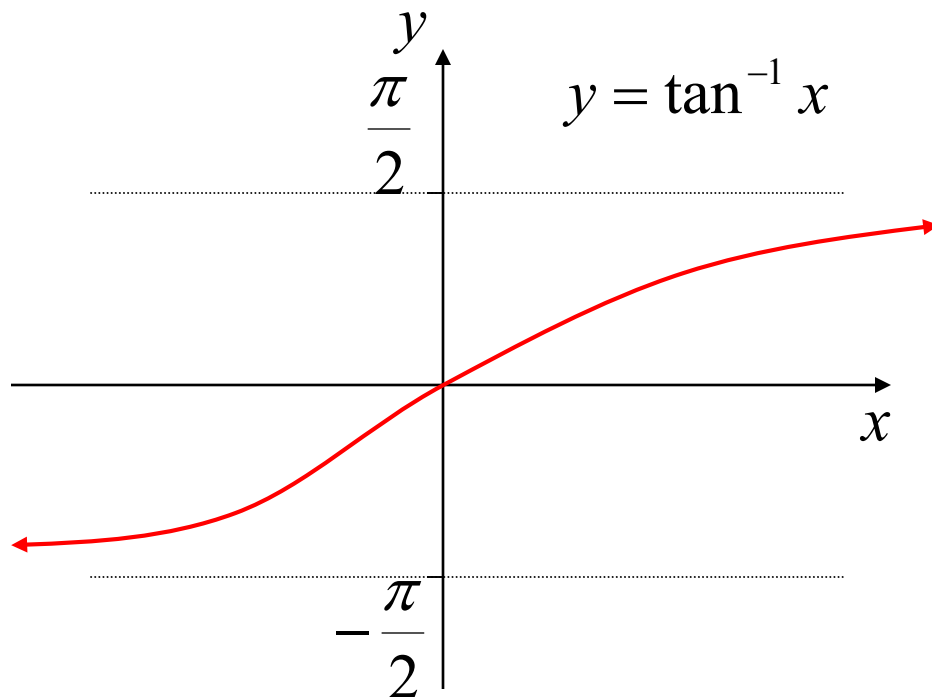
Restricted Domain:  $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Range: all real  $y$

$f^{-1} : y = \tan^{-1} x$

Domain: all real  $x$

Range:  $-\frac{\pi}{2} < y < \frac{\pi}{2}$

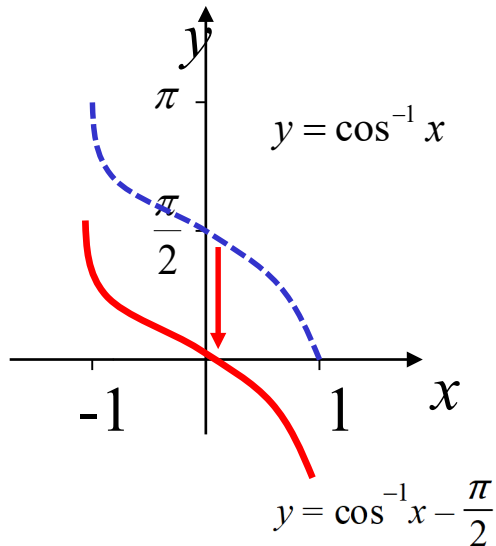


$$\sin^{-1}(-x) = -\sin^{-1} x \quad (\text{odd function})$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \quad \left( \text{odd function shifted } \uparrow \frac{\pi}{2} \right)$$

$$\tan^{-1}(-x) = -\tan^{-1} x \quad (\text{odd function})$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad (\angle \text{ sum } \Delta)$$



$$f(-x) = -f(x)$$

$$\cos^{-1}(-x) - \frac{\pi}{2} = - \left\{ \cos^{-1} x - \frac{\pi}{2} \right\}$$

$$\cos^{-1}(-x) - \frac{\pi}{2} = -\cos^{-1} x + \frac{\pi}{2}$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

### *Alternative notations*

$$\sin^{-1} x = \arcsin x = \text{asin} x$$

e.g. Find the exact value of;

$$(i) \tan^{-1} \sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \underline{\frac{\pi}{12}}$$

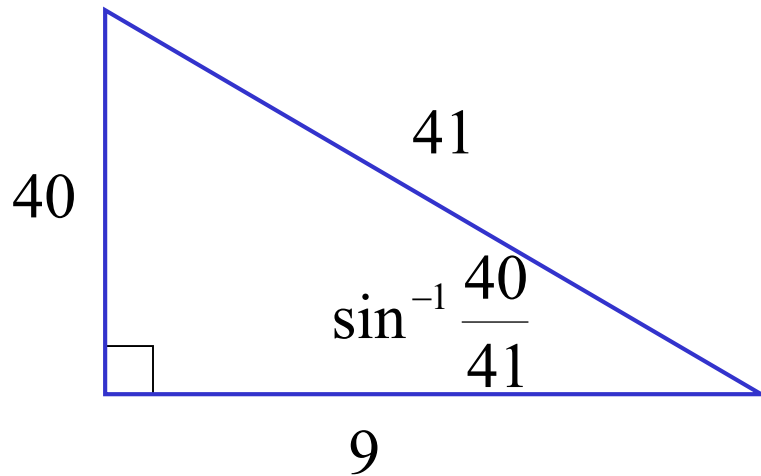
$$(ii) \sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} \left( -\frac{1}{2} \right)$$

$$= \frac{\pi}{4} - \left( -\frac{\pi}{6} \right)$$

$$= \underline{\frac{5\pi}{12}}$$

$$(iii) \cos \sin^{-1} \frac{40}{41}$$

$$= \underline{\frac{9}{41}}$$



$$(iv) \sin^{-1} \sin \frac{5\pi}{6} = \underline{\frac{\pi}{6}}$$

**Exercise 17B; 1 to 5 ace etc, 7, 8, 11, 12, 13, 14**