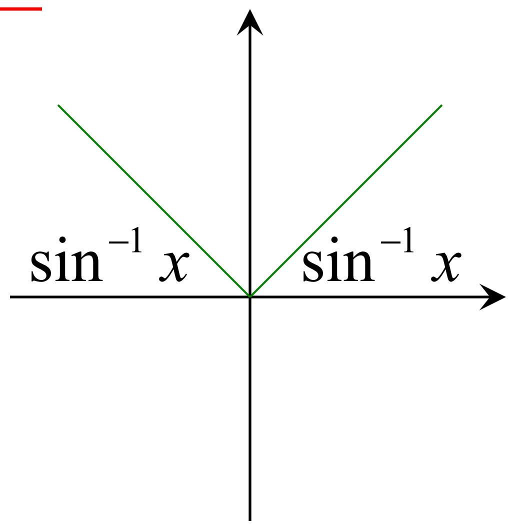
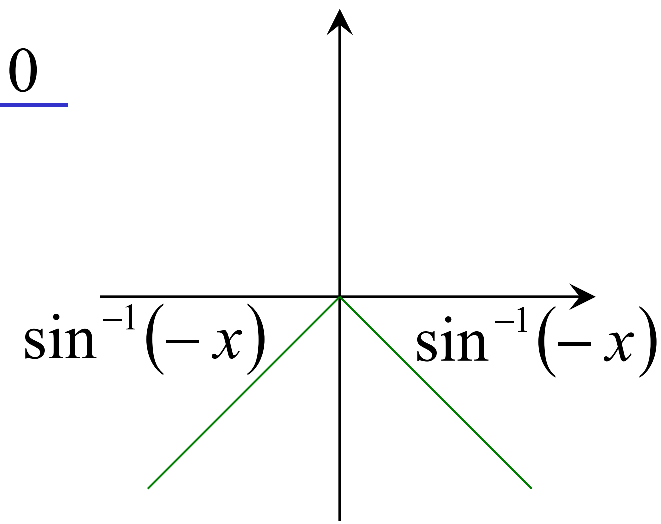
General Solutions of Trig Equations

$\sin\theta = x$

$x > 0$



$x < 0$



$\theta = \sin^{-1} x \text{ or } \pi - \sin^{-1} x$

$\theta = \pi + \sin^{-1}(-x) \text{ or } 2\pi - \sin^{-1}(-x)$

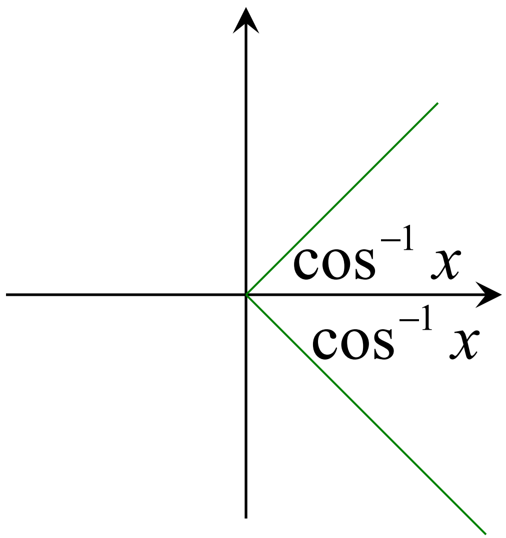
$\theta = \pi - \sin^{-1} x \text{ or } 2\pi + \sin^{-1} x$

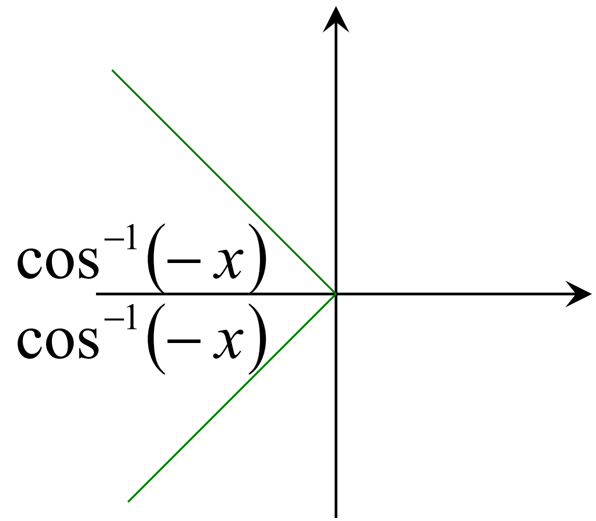
$\sin\theta = x$

$\theta = \pi k + (-1)^k \sin^{-1} x \quad \text{where } k \in \mathbb{Z}$

$\cos\theta = x$

$x > 0$ $\qquad\qquad\qquad\qquad$ $x < 0$





$$\theta = \cos^{-1} x \text{ or } 2\pi - \cos^{-1} x$$

$$\theta = \pi - \cos^{-1}(-x) \text{ or } \pi + \cos^{-1}(-x)$$

$$\theta = \pi - \left(\pi - \cos^{-1} x\right) \text{ or } \pi + \left(\pi - \cos^{-1} x\right)$$

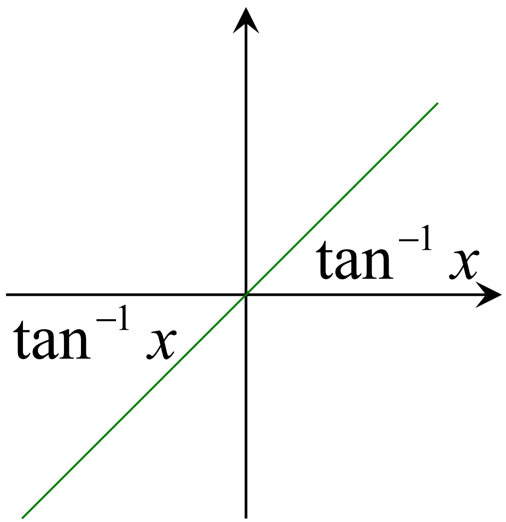
$$= \cos^{-1} x \qquad\qquad \text{or } 2\pi - \cos^{-1} x$$

$$\cos\theta = x$$

$$\theta = 2\pi k \pm \cos^{-1} x \qquad \text{where } k \in \mathbb{Z}$$

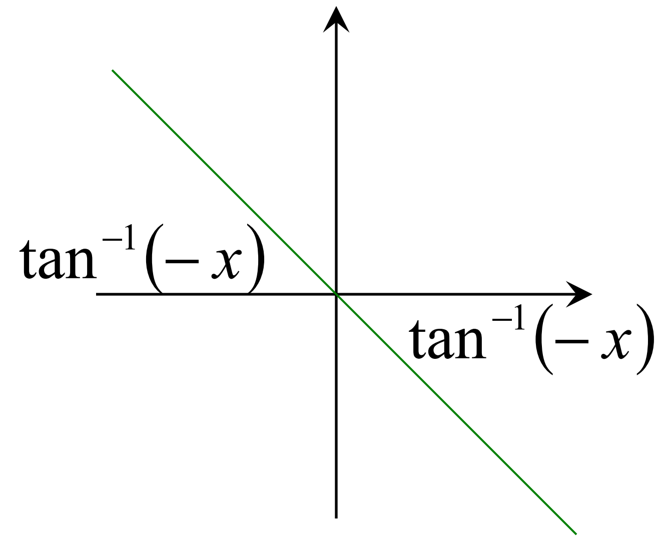
$\tan\theta = x$

$x > 0$



$$\theta = \tan^{-1} x \text{ or } \pi + \tan^{-1} x$$

$x < 0$



$$\theta = \pi - \tan^{-1}(-x) \text{ or } 2\pi - \tan^{-1}(-x)$$

$$\theta = \pi + \tan^{-1} x \text{ or } 2\pi + \tan^{-1} x$$

$$\tan\theta = x$$

$$\theta = \pi k + \tan^{-1} x \qquad \text{where } k \in \mathbb{Z}$$

e.g. $(i)\sin\theta = \frac{\sqrt{3}}{2}$

$$\theta = \pi k + (-1)^k \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

where $k \in \mathbb{Z}$

$$\theta = \pi k + (-1)^k \frac{\pi}{3}$$

OR

$$\sin\theta = \frac{\sqrt{3}}{2}$$

Q1, Q2

$$\theta = \frac{\pi}{3} + 2\pi k \quad \text{or} \quad \frac{2\pi}{3} + 2\pi k$$

where $k \in \mathbb{Z}$

$(ii)\cos\theta = -\frac{1}{\sqrt{2}}$

$$\theta = 2\pi k \pm \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

where $k \in \mathbb{Z}$

$$\theta = 2\pi k \pm \frac{3\pi}{4}$$

OR

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

Q2, Q3

$$\theta = \frac{3\pi}{4} + 2\pi k \quad \text{or} \quad \frac{5\pi}{4} + 2\pi k$$

where $k \in \mathbb{Z}$

$$(iii)\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \pi k + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

where $k \in \mathbb{Z}$

$$\theta = \pi k + \frac{\pi}{6}$$

OR

$$\tan\theta = \frac{1}{\sqrt{3}}$$

Q1

$$\theta = \frac{\pi}{6} + \pi k \quad \text{where } k \in \mathbb{Z}$$

Only one solution is required as the period of tan is π

$$(iv)\sin\theta = \sin\frac{5\pi}{7}$$

$$\theta = \pi k + (-1)^k \sin^{-1}\sin\frac{5\pi}{7}$$

$$\theta = \pi k + (-1)^k \frac{2\pi}{7}$$

where $k \in \mathbb{Z}$

inverse sine can only produce acute or negative acute angles

$$(v)\cos 2x = \cos\frac{\pi}{9}$$

$$2x = 2\pi k \pm \cos^{-1}\cos\frac{\pi}{9}$$

$$2x = 2\pi k \pm \frac{\pi}{9}$$

$$x = \pi k \pm \frac{\pi}{18} \quad \text{where } k \in \mathbb{Z}$$

(vi) $\cos 4x = \cos x$

$$4x = 2\pi k \pm \cos^{-1} \cos x \quad \text{where } k \in \mathbb{Z}$$

if $x > 0$; $4x = 2\pi k \pm x$

if $x < 0$; $4x = 2\pi k \pm x$

$$4x \mp x = 2\pi k$$

$$3x = 2\pi k \quad \text{or} \quad 5x = 2\pi k$$

$$x = \frac{2\pi k}{3} \qquad x = \frac{2\pi k}{5}$$

cosine is an even function
so;
$\cos x = \cos(-x)$
thus
$\cos^{-1}\cos x = \cos^{-1}\cos(-x)$
$= -x$ as $x < 0$
so there is no need
to consider cases

$$\therefore x = \frac{2\pi k}{3} \text{ or } \frac{2\pi k}{5}, \text{ where } k \in \mathbb{Z}$$

(vii) $\sin 4x = \sin x$

$$4x = \pi k + (-1)^k \sin^{-1} \sin x \qquad \text{where } k \in \mathbb{Z}$$

if $x > 0$; $4x = \pi k + (-1)^k x$

if k is odd ;

$$4x = \pi k - x$$

$$5x = \pi k$$

$$x = \frac{\pi k}{5}$$

$$x = \frac{\pi(2n+1)}{5} \quad n \in \mathbb{Z}$$

if k is even ;

$$4x = \pi k + x$$

$$3x = \pi k$$

$$x = \frac{\pi k}{3}$$

$$x = \frac{2\pi n}{3} \quad n \in \mathbb{Z}$$

no need to check the positive and negative cases for inverse sine

if $x < 0$; $\sin^{-1} \sin x = x$ i.e. the same results as when $x > 0$

$$\therefore x = \frac{\pi(2k+1)}{5} \text{ or } \frac{2\pi k}{3}, \quad \text{where } k \in \mathbb{Z}$$

general solutions handout;
1, 4bdf, 5ace, 6acd, 7acehjk,
8ace, 9, 10, 11, 12ad, 13, 14