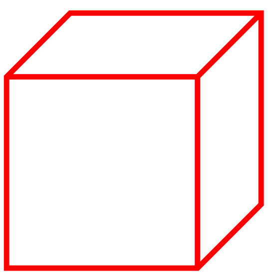
Rates of Change

In some cases two, or more, rates must be found to get the equation in terms of the given variable.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

e.g. (i) A block of ice in the form of a cube has one edge 10 cm long. It is melting so that its dimensions decrease at the rate of 1 mm/s.

At what rate is the volume decreasing when the edge is 5cm long?



$$\frac{dV}{dt} = ? \qquad \frac{dx}{dt} = -\frac{1}{10}$$

$$V = x^3$$

$$\frac{dV}{dx} = 3x^2$$

$$\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$$

$$= 3x^2 \times -\frac{1}{10}$$

$$= -\frac{3x^2}{10}$$

$$\text{when } x = 5, \frac{dV}{dt} = -\frac{3(5)^2}{10}$$

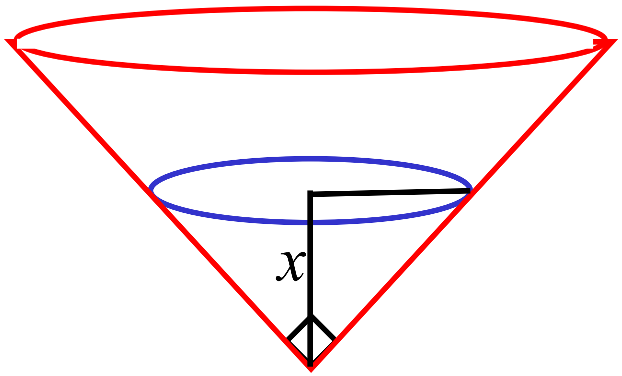
$$= -7.5$$

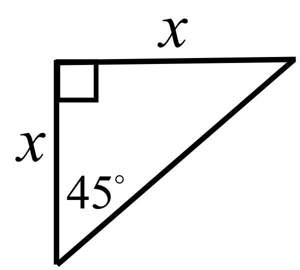
$\therefore$ volume is decreasing at 7.5 cm^3/s

(ii) A vessel is in the form of an inverted cone with a vertical angle of 90°

If the depth of the water in the vessel is x cm;

a) find the volume of water.





$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi x^2 x$$
$$= \frac{1}{3}\pi x^3$$

b) If water is poured in at a rate of 0.2 cm^3/min, find the rate the depth is increasing when the water depth is 4 cm.

$$\frac{dx}{dt} = ?$$
$$\frac{dV}{dt} = \frac{1}{5}$$
$$V = \frac{1}{3}\pi x^3$$
$$\frac{dV}{dx} = \pi x^2$$

$$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$$
$$= \frac{1}{5} \times \frac{1}{\pi x^2}$$
$$= \frac{1}{5\pi x^2}$$

when $x = 4, \dfrac{dx}{dt} = \dfrac{1}{5\pi(4)^2}$

$$= \frac{1}{80\pi}$$

$\therefore$ depth is increasing at $\dfrac{1}{80\pi}$ cm/min

(iii) A spherical bubble is expanding so that its volume increases at a constant rate of $70\text{mm}^3/\text{s}$

What is the rate of increase of its surface area when the radius is 10 mm?

$$\frac{dS}{dt} = ? \qquad \frac{dV}{dt} = 70 \qquad V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$$

$$\frac{dV}{dr} = 4\pi r^2 \qquad \frac{dS}{dr} = 8\pi r$$

$$\frac{dS}{dt} = \frac{dV}{dt} \cdot \frac{dS}{dr} \cdot \frac{dr}{dV}$$

$$= (70)(8\pi r)\left(\frac{1}{4\pi r^2}\right)$$

$$= \frac{140}{r}$$

$$\text{when } r = 10, \frac{dV}{dt} = \frac{140}{10} = 14$$

$\therefore$ when radius is 10mm the surface area is increasing at a rate of $14\text{mm}^2/\text{s}$

Exercise 16A; 1a, 2a, 4, 6, 7, 8, 9, 10, 13, 15, 16, 18