Products to Sums

$2\cos A\cos B = \cos(A-B) + \cos(A+B)$ from $\cos(\alpha+\beta)$

$2\sin A\cos B = \sin(A-B) + \sin(A+B)$ from $\sin(\alpha+\beta)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$ from $\cos(\alpha+\beta)$

eg (i) Express as a sum or difference of trig functions

a) $2\cos 5x\sin x = 2\sin x\cos 5x$

$= \sin(x-5x) + \sin(x+5x)$

$= \sin(-4x) + \sin 6x$

$= -\sin 4x + \sin 6x$

b) $\cos 3\theta\cos 5\theta = \frac{1}{2}(2\cos 3\theta\cos 5\theta)$

$= \frac{1}{2}(\cos(3\theta+5\theta)+\cos(3\theta-5\theta))$

$= \frac{1}{2}(\cos 8\theta+\cos(-2\theta)) = \frac{1}{2}(\cos 8\theta+\cos 2\theta)$

(ii) Evaluate $2\sin 45^\circ \cos 15^\circ$

$$= \sin(45° - 15°) + \sin(45° + 15°)$$
$$= \sin 30° + \sin 60°$$
$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$
$$= \frac{1 + \sqrt{3}}{2}$$

(*iii*) Prove $\sin^2 5\theta - \sin^2 3\theta = \sin 8\theta \sin 2\theta$

$$\sin 8\theta \sin 2\theta = \frac{1}{2}\{\cos(8\theta - 2\theta) - \cos(8\theta + 2\theta)\}$$
$$= \frac{1}{2}(\cos 6\theta - \cos 10\theta)$$
$$= \frac{1}{2}(1 - 2\sin^2 3\theta - 1 + 2\sin^2 5\theta)$$
$$= \frac{1}{2}(2\sin^2 5\theta - 2\sin^2 3\theta)$$
$$= \sin^2 5\theta - \sin^2 3\theta$$

Sums to Products

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$ *(2 sine half sum cos half diff)*

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$ *(2 cos half sum cos half diff)*

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$ *(minus 2 sine half sum sine half diff)*

e.g. (*i*) Prove $\dfrac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \sqrt{3}$

$$\frac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \frac{\cos 75° + \cos 15°}{\sin 75° + \sin(-15°)}$$

$$= \frac{2\cos\frac{1}{2}(90°)\cos\frac{1}{2}(60°)}{2\sin\frac{1}{2}(60°)\cos\frac{1}{2}(90°)} = \frac{\cos 30°}{\sin 30°} = \cot 30°$$

$$= \sqrt{3}$$

(ii) Solve $\sin x + \sin 3x = 0$ $\quad 0^\circ \le x \le 360^\circ$

$$2\sin 2x \cos(-x) = 0$$

$$2\sin 2x \cos x = 0$$

$$\sin 2x = 0 \quad \text{or} \quad \cos x = 0$$

$$2x = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ \qquad x = 90^\circ, 270^\circ$$

$$x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$$

$$\therefore x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$$

Exercise 17G; 1b i, ii, v, vii, 3b, 4b, 5a, 6b, 7, 9, 11bc, 12ac, 13b