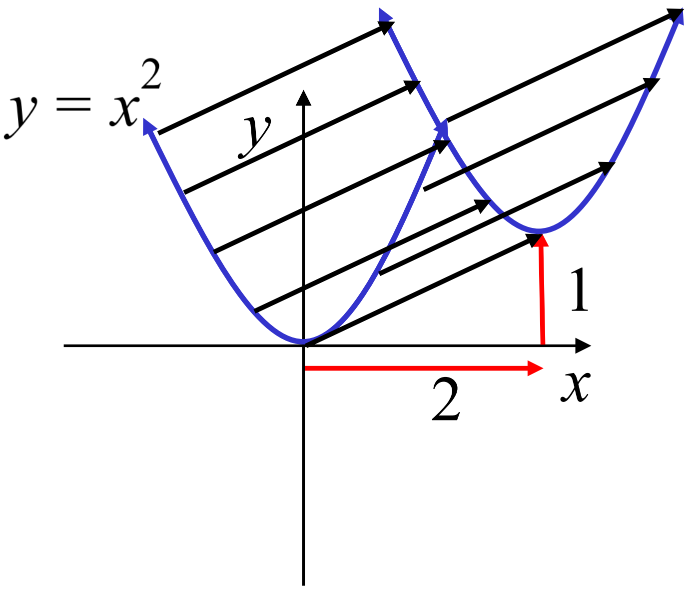
Introduction to Vectors

Translation of Graphs

e.g. The parabola $y = x^2$ is translated 2 units to the right and 1 unit up

Find the new equation of the parabola

$$y = (x - 2)^2 + 1$$



$$x^2 \rightarrow (x - 2)^2 \rightarrow (x - 2)^2 + 1$$

We found the new equation using a horizontal and a vertical translation

These two translations can be combined into one **displacement vector**

Every point on the parabola can be moved into its new position by sliding along the same displacement vector

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

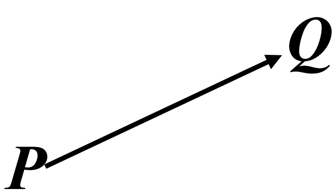
Using parametrics; $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} t \\ t^2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} t + 2 \\ t^2 + 1 \end{pmatrix}$$

$X = t + 2$ $\qquad$ $Y = t^2 + 1$

$t = X - 2$ $\qquad$ $Y = (X - 2)^2 + 1$

Definitions



$\overrightarrow{PQ}$ is the **displacement** vector joining P to Q

P is the **tail** of the vector

Q is the **head** of the vector

The other notations for a vector are **p** or $\underset{\sim}{p}$

All vectors can be uniquely identified by its length (**magnitude**) and the angle it makes with the horizontal (**direction**)

$$\left|\underset{\sim}{p}\right| = \left|\overrightarrow{\mathrm{PQ}}\right| = \text{ magnitude of vector}$$

Opposite (negative) vectors are directed in opposite directions

$$\text{i.e. if } \overrightarrow{PQ} = \underset{\sim}{p} \text{ then } \overrightarrow{QP} = -\underset{\sim}{p}$$

Parallel vectors have the same slope

$$\text{i.e. } \overrightarrow{PQ} \parallel \overrightarrow{QP}$$

Zero vector, $\overrightarrow{PP}$, is the single point P, it is the only vector to have no magnitude and no direction and is thus parallel to all vectors.

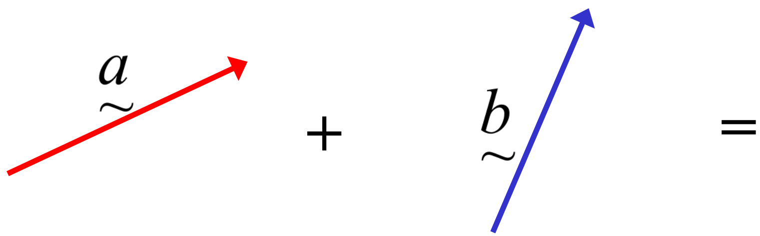
Scalar is a magnitude without direction i.e. its just a real number

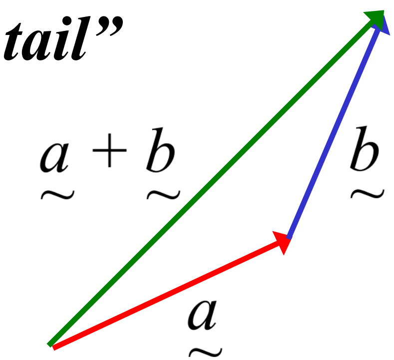
$\lambda \underset{\sim}{p}$ where λ is a scalar, is a vector with direction of $\underset{\sim}{p}$ and length $\lambda \left| \underset{\sim}{p} \right|$

Position vector is the displacement vector whose tail is the origin

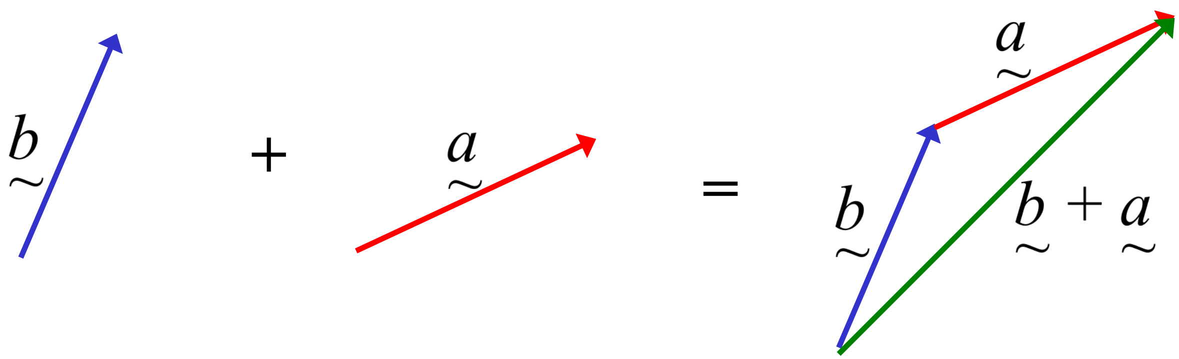
Addition and Subtraction of Vectors

To add two vectors, place them ***“head to tail”***



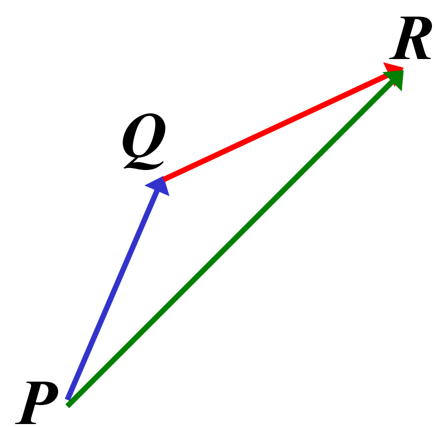


Addition of vectors is commutative

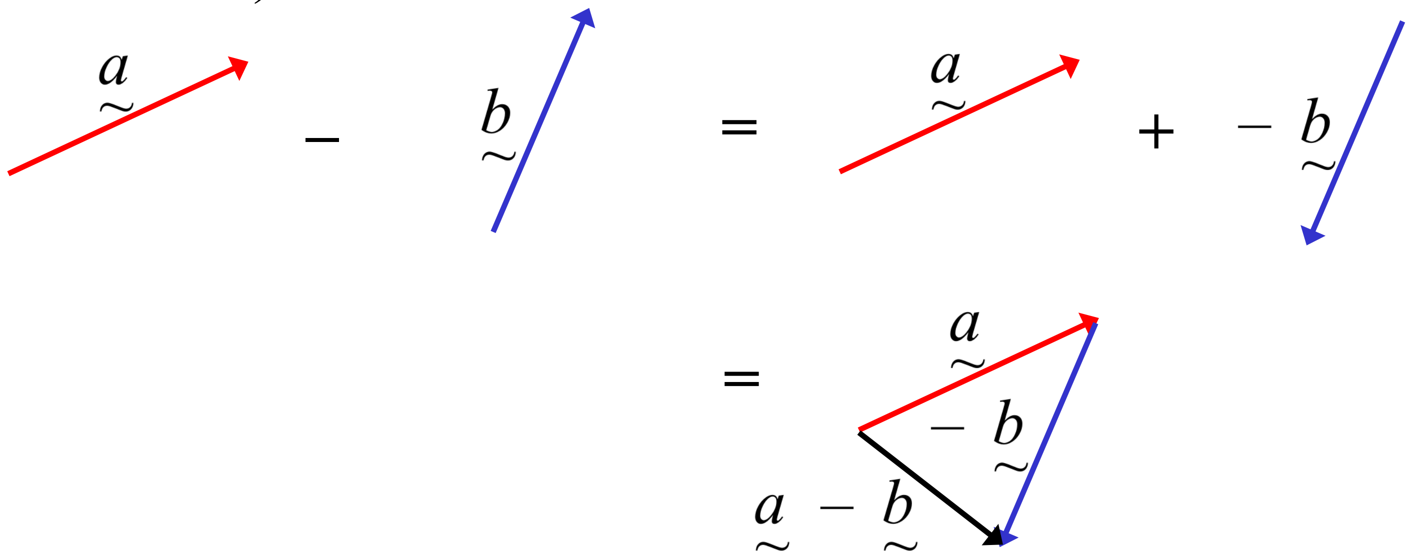


Another way of expressing this would be

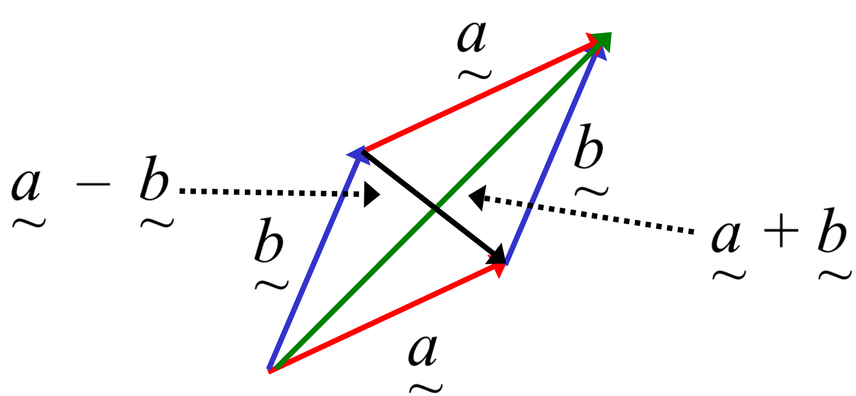
$$\boxed{\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}}$$



To **subtract** two vectors, place the vectors ***"head to head"*** (or add the negative vector)

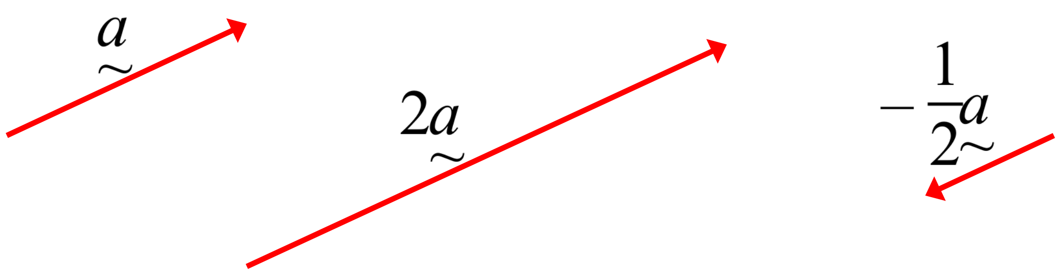


NOTE: the vectors $\underset{\sim}{a} + \underset{\sim}{b}$ *and* $\underset{\sim}{a} - \underset{\sim}{b}$ *are the diagonals of the parallelogram created by the vectors* $\underset{\sim}{a}$ *and* $\underset{\sim}{b}$



Multiplication of Vectors by Scalars

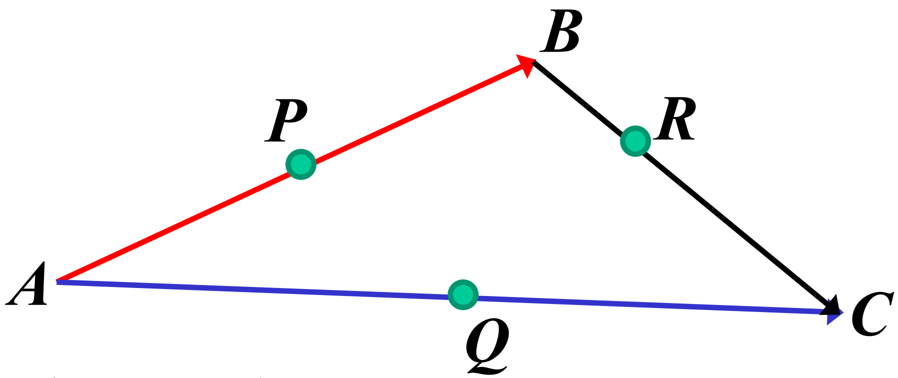
Multiplying a vector by a scalar, λ enlarges the magnitude of the vector by a factor of λ



NOTE: all of these vectors are parallel e.g. ΔABC is a triangle with $\overrightarrow{AB} = a$ and $\overrightarrow{AC} = c$

P and Q are midpoints of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively

R is a point on $\overrightarrow{BC}$ such that $\overrightarrow{RC} = 2 \times \overrightarrow{BR}$



(i) Express $\overrightarrow{BC}$ and $\overrightarrow{PQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

$$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} \quad \textit{(head minus tail)}$$
$$= \underset{\sim}{c} - \underset{\sim}{a}$$

$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB}$$
$$= \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$$

(ii) Compare the vectors $\overrightarrow{BC}$ and $\overrightarrow{PQ}$

$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{BC}$$

(*iii*) What geometric property of a triangle does (*ii*) demonstrate?

The line joining the midpoints of two sides of a triangle is parallel to the third side (and half its length)

(*iv*) Express the vectors $\overrightarrow{BR}$ and $\overrightarrow{RC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

$$\overrightarrow{BR} = \frac{1}{3}\overrightarrow{BC}$$
$$= \underline{\frac{1}{3}(\underset{\sim}{c} - \underset{\sim}{a})}$$

$$\overrightarrow{RC} = \frac{2}{3} \times \overrightarrow{BC}$$
$$= \underline{\frac{2}{3}(\underset{\sim}{c} - \underset{\sim}{a})}$$

(*v*) Show that $\overrightarrow{AR} = \frac{1}{3}(2\underset{\sim}{a} + \underset{\sim}{c})$

$$\overrightarrow{AR} = \overrightarrow{AB} + \overrightarrow{BR}$$
$$= \underset{\sim}{a} + \frac{1}{3}(\underset{\sim}{c} - \underset{\sim}{a})$$
$$= \underset{\sim}{a} + \frac{1}{3}\underset{\sim}{c} - \frac{1}{3}\underset{\sim}{a}$$
$$= \frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{c} = \underline{\frac{1}{3}(2\underset{\sim}{a} + \underset{\sim}{c})}$$

Exercise 8A; 1a, 2, 5a, 7, 8, 9, 10, 11, 13, 15, 16, 17, 20