Sum Of A Geometric Series

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$rS_n = \qquad ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$$

$$(r-1)S_n = ar^n - a$$

$$S_n = \frac{a(r^n - 1)}{r-1} \qquad ,\text{if } |r| > 1$$

$$OR$$

$$S_n = \frac{a(1 - r^n)}{1-r} \qquad ,\text{if } |r| < 1$$

$e.g.(i)$ Find the sum of the first 10 terms of $2+6+18+\dots$

$a=2, r=3$ and $n=10$

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

$$S_{10}=\frac{2\left(3^{10}-1\right)}{3-1}$$

$$=\underline{59048}$$

$(ii)\ \sum_{n=3}^{8} 6\left(\frac{1}{2}\right)^{n-1}$

$$a=6\left(\frac{1}{2}\right)^2 \qquad r=\frac{1}{2}, n=6$$

$$=\frac{3}{2}$$

$$S_n=\frac{a\left(1-r^n\right)}{1-r}$$

$$S_6=\frac{\frac{3}{2}\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}}$$

$$=\frac{3}{2}\times\frac{63}{64}\times\frac{2}{1}$$

$$=\underline{\frac{189}{64}}$$

(iii) **2021 Advanced HSC Q34**

A discrete random variable has probability distribution as shown in the table where n is a finite positive integer.

x	r	r^2	r^3	$\dots$	r^k	$\dots$	r^n
$P(X=x)$	r^n	r^{n-1}	r^{n-2}	$\dots$	r^{n-k+1}	$\dots$	r

Show that $E(X) = n(2r - 1)$.

x	r	r^2	r^3	$\dots$	r^k	$\dots$	r^n
$P(X=x)$	r^n	r^{n-1}	r^{n-2}	$\dots$	r^{n-k+1}	$\dots$	r
$xp(x)$	r^{n+1}	r^{n+1}	r^{n+1}	$\dots$	r^{n+1}	$\dots$	r^{n+1}

$$E(X) = \Sigma xp(x)$$
$$= r^{n+1} + r^{n+1} + r^{n+1} + \dots + r^{n+1}$$
$$= nr^{n+1}$$
$$= n(2r-1)$$

$$\Sigma P(X = x) = 1$$
$$r^n + r^{n-1} + r^{n-2} + \dots r = 1$$
$$\frac{r(1-r^n)}{1-r} = 1$$
$$r - r^{n+1} = 1 - r$$
$$r^{n+1} = 2r - 1$$

Exercise 1G;
4cf, 7, 8b, 9c, 12,
13, 14, 16a, 18, 20b