Dilations

horizontal dilation (stretch) replace x with $\frac{x}{a}$

$$y = f\left(\frac{x}{a}\right)$$

curve is stretched horizontally by a factor of a

(if $a > 1$, curve is shallower)

domain may be altered, range will be unchanged

vertical dilation (stretch) replace y with $\frac{y}{a}$

$$\frac{y}{a} = f(x) \quad \textbf{\textit{OR}} \quad y = af(x)$$

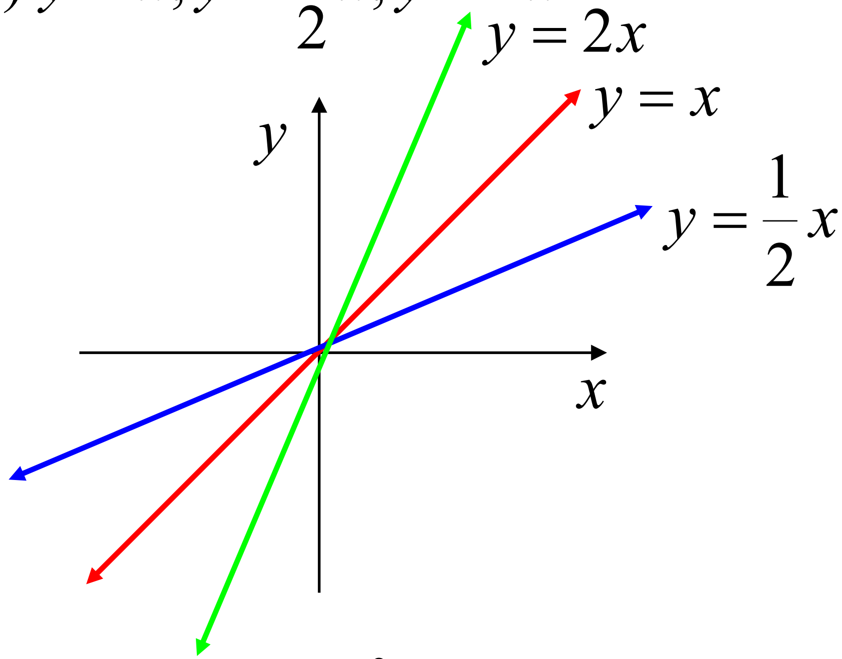
curve is stretched vertically by a factor of a

(if $a > 1$, curve is steeper)

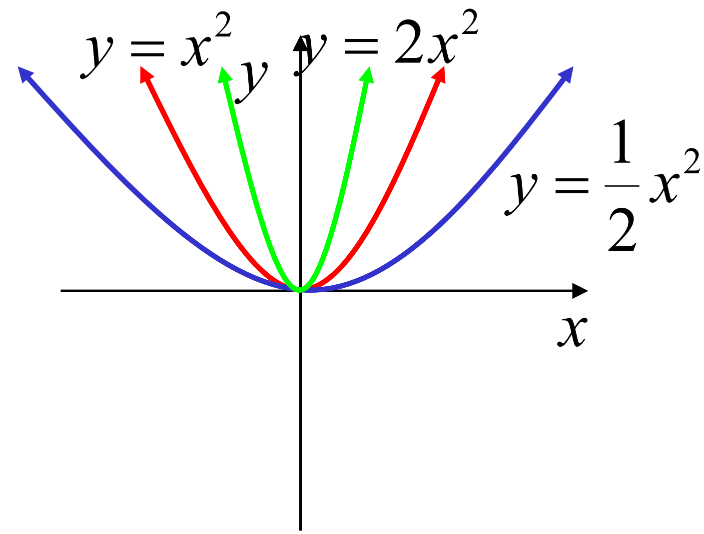
domain unchanged, range may be altered

e.g. (i) on one graph draw

$a)\ y = x, y = \frac{1}{2}x, y = 2x$



$b)\ y = x^2, y = \frac{1}{2}x^2, y = 2x^2$

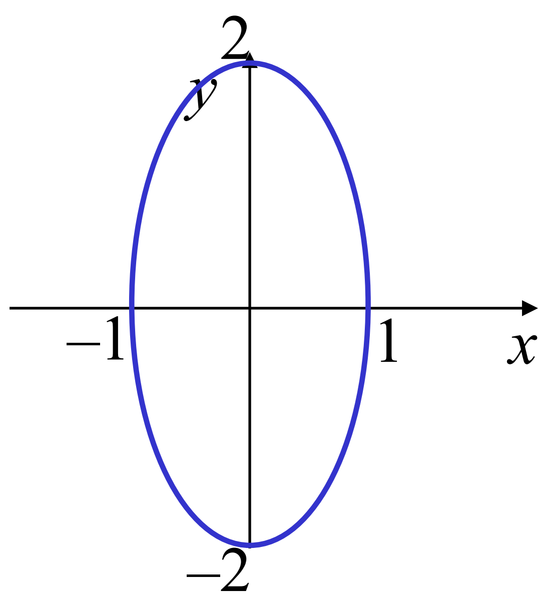


(ii) Sketch $x^2 + \frac{y^2}{4} = 1$

1. *basic curve*: $x^2 + y^2 = 1$

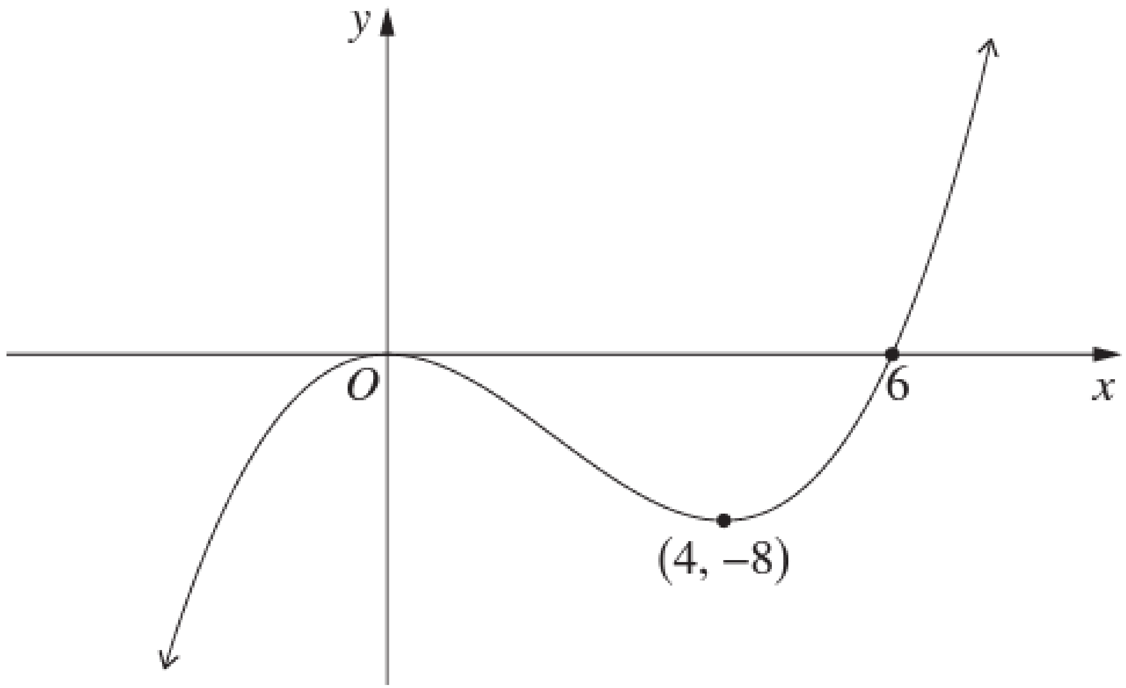
2. $\frac{y^2}{4} = \left(\frac{y}{2}\right)^2, \therefore k = 2$

stretch vertically by a factor of 2



(*iii*) **2021 Advanced HSC Q21**

Consider the graph of $y = f(x)$ as shown.

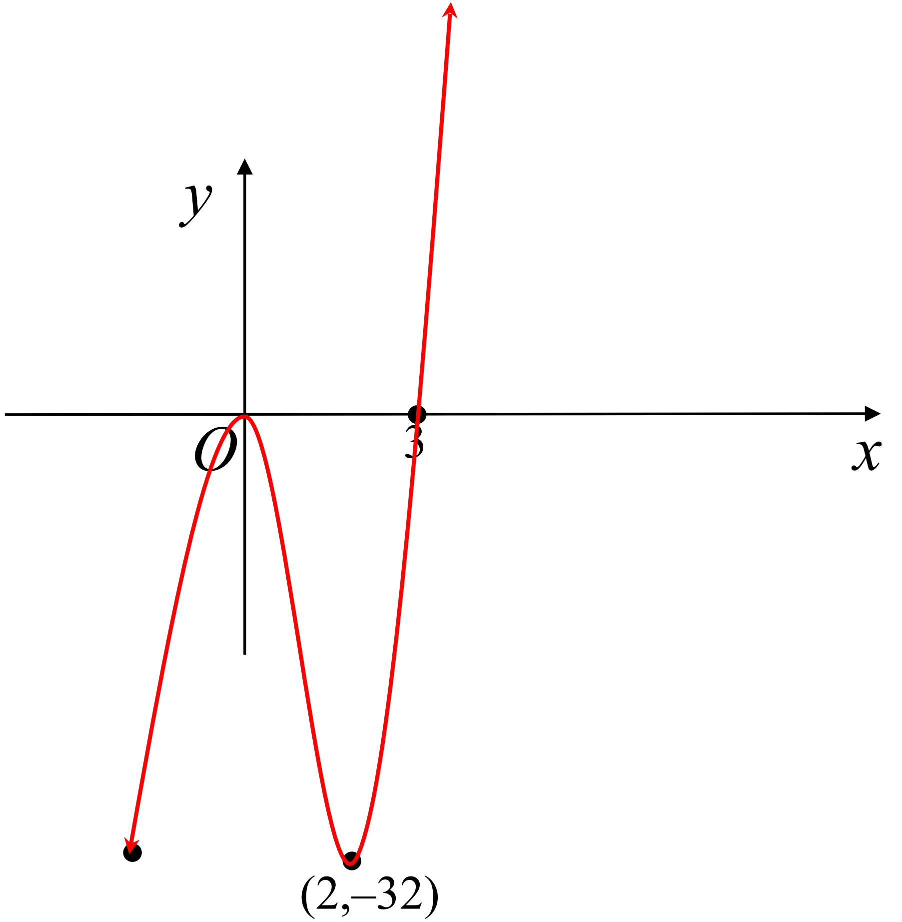


Sketch the graph of $y = 4f(2x)$ showing x-intercepts and the coordinates of the turning points.

$$y = 4f(2x) \Rightarrow \frac{y}{4} = f\left(\frac{x}{\frac{1}{2}}\right)$$

1. Stretch horizontally by a factor of $\frac{1}{2}$

2. Stretch vertically by a factor of 4



Enlargements

An enlargement is when the **same** dilation factor is applied both horizontally and vertically.

$$f(x,y) \Rightarrow f\left(\frac{x}{a}, \frac{y}{a}\right)$$

e.g. The circle $(x-1)^2 + (y+2)^2 = 1$ is enlarged by a factor of 2. Using the origin as the centre of enlargement, find the circle's new equation

$$(x-1)^2 + (y+2)^2 = 1 \Rightarrow \left(\frac{x}{2} - 1\right)^2 + \left(\frac{y}{2} + 2\right)^2 = 1$$

$$\underline{(x-2)^2 + (y+4)^2 = 4}$$

Not all transformations commute

A mathematical operation **commutes** if the order of the objects being operated on does not matter

addition and multiplication commute

$3 + 4 = 4 + 3$

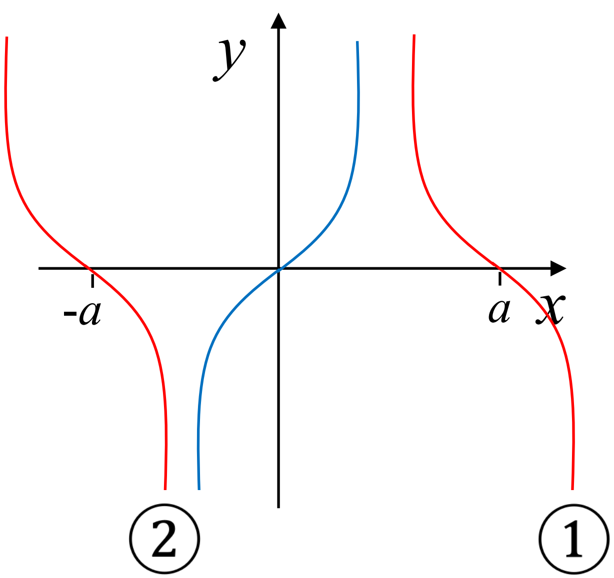
$3 \times 4 = 4 \times 3$

subtraction and division do not commute

$3 - 4 \neq 4 - 3$

$3 \div 4 \neq 4 \div 3$

Transformations can be done in any order, with the exception of a dilation and a translation in the **same** direction



① reflect in $x = 0$ then shift right

reflection is a dilation of factor – 1

② shift right then reflect in $x = 0$

e.g. Determine the equation after $y = x^2$ has been;

(**i**) shifted up 1 unit then reflected vertically

$$x^2 \rightarrow x^2 + 1 \rightarrow -(x^2 + 1) \quad \Rightarrow \underline{y = -x^2 - 1}$$

(**ii**) reflected vertically the shifted up 1 unit

$$x^2 \rightarrow -x^2 \rightarrow -x^2 + 1 \quad \Rightarrow \underline{y = 1 - x^2}$$

Exercise 3H; 1aceg, 2bd, 3, 4, 5a, 6a, 8, 9ad, 10, 13, 14, 15a, 17

Exercise 3I; 1 to 5, 6bdfh, 8ab, 12