Complex Numbers

Solving Quadratics

$$x^2 + 1 = 0$$

$$x^2 = -1$$

no real solutions

In order to solve this equation we define a new number

$$i = \sqrt{-1} \quad \text{or} \quad i^2 = -1$$

i is an imaginary number

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

$$x = \pm i$$

All complex numbers (z) can be written as; $z = x + iy$

Definitions;

(1) All complex numbers contain a **real** and an **imaginary** part

$$\mathrm{Re}(z) = x$$
$$\mathrm{Im}(z) = y$$

$e.g.\ z = 3 + 5i$

$\mathrm{Re}(z) = 3 \qquad \mathrm{Im}(z) = 5$

(2) If Re(z) = 0, then z is an **imaginary number**

$e.g.\ \sqrt{3}i, -6i$

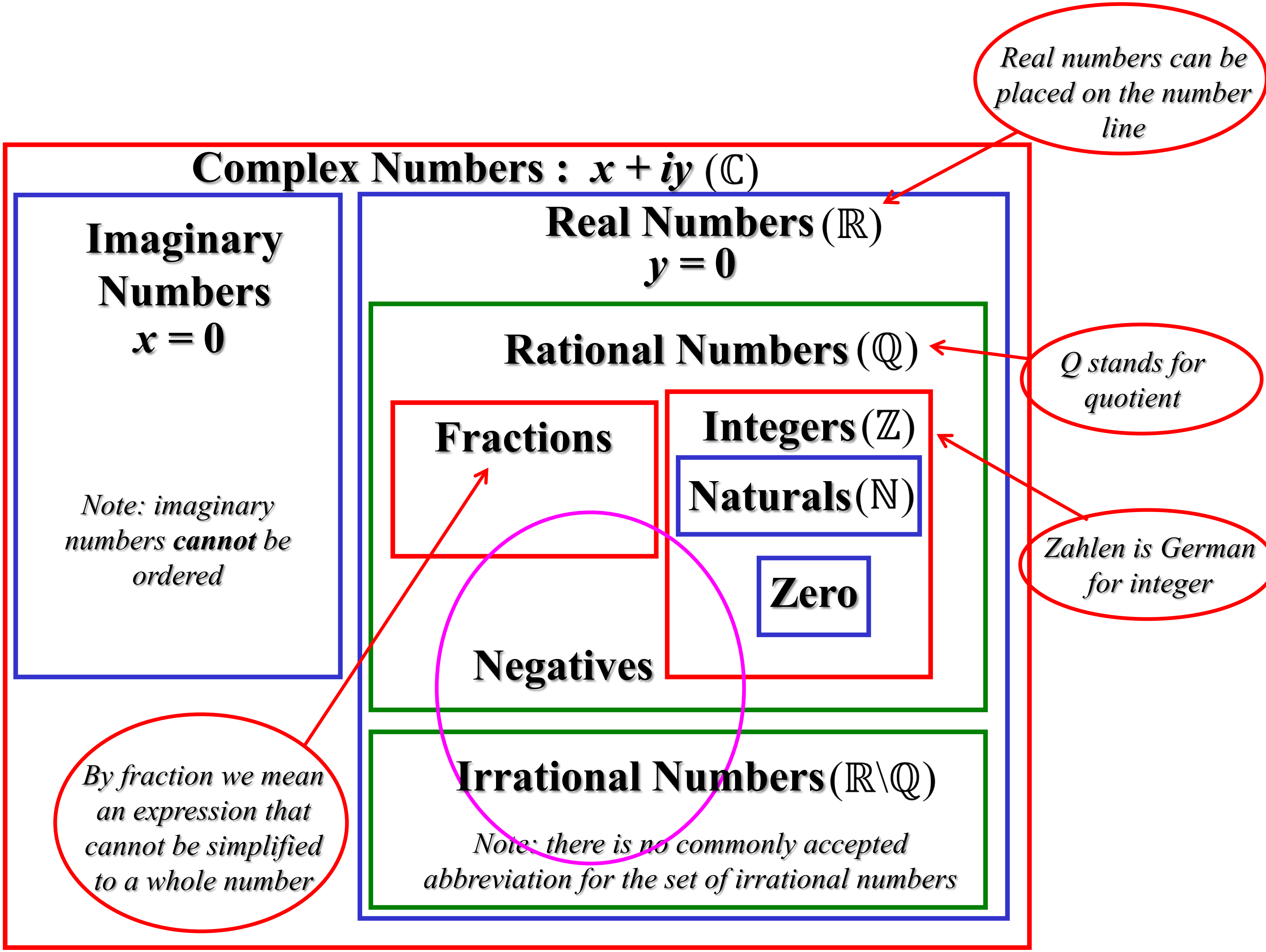
(3) If Im(z) = 0, then z is a **real number**

$e.g.\ \frac{3}{4}, \pi, e, -4$

(4) Every complex number $z = x + iy$, has a **complex conjugate**

$$\bar{z} = x - iy$$

$e.g.\ z = -2 - \sqrt{7}i \qquad \bar{z} = -2 + \sqrt{7}i$



Real numbers can be placed on the number line
Complex Numbers : $x + iy$ ($\mathbb{C}$)
Imaginary Numbers $x = 0$
Note: imaginary numbers **cannot** be ordered
Real Numbers ($\mathbb{R}$) $y = 0$
Rational Numbers ($\mathbb{Q}$)
Q stands for quotient
Fractions
Integers ($\mathbb{Z}$)
Naturals ($\mathbb{N}$)
Zero
Zahlen is German for integer
Negatives
By fraction we mean an expression that cannot be simplified to a whole number
Irrational Numbers ($\mathbb{R} \backslash \mathbb{Q}$)
Note: there is no commonly accepted abbreviation for the set of irrational numbers

Basic Operations

As i a surd, the operations with complex numbers are the same as surds

Addition

$$(4-3i)+(-8+2i)$$

$$=-4-i$$

Subtraction

$$(4-3i)-(-8+2i)$$

$$=12-5i$$

Multiplication

$$(4-3i)(-8+2i)$$

$$=-32+8i+24i-6i^2$$

$$=-32+32i+6$$

$$=-26+32i$$

Division (Realising The Denominator)

$$\frac{(4-3i)}{(-8+2i)} \times \frac{(-8-2i)}{(-8-2i)}$$

$$=\frac{-32-8i+24i-6}{64+4}$$

$$=\frac{-38+16i}{68}$$

$$=\frac{-19}{34}+\frac{8}{34}i$$

Conjugate Basics

$$(1)\ \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$(2)\ \overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$$

$$(3)\ \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

$$(4)\ z\overline{z} = x^2 + y^2$$

$$(5)\ \frac{1}{z} = \frac{\overline{z}}{z\overline{z}}$$

$$\frac{1}{(4-3i)} = \frac{4+3i}{16+9}$$

$$= \frac{4}{25} + \frac{3}{25}i$$

Complex Equations

If two complex numbers are equal, then their real parts are equal and their imaginary parts are equal

$$i.e.\ \text{If } a_1 + b_1 i = a_2 + b_2 i$$

then

$$a_1 = a_2$$
$$b_1 = b_2$$

$e.g.$ (i) $(2+3i)(x+iy) = (4-2i)$

$$2x + 2iy + 3ix - 3y = 4 - 2i$$

$$\therefore 2x - 3y = 4 \text{ and } 3x + 2y = -2$$

$$\Rightarrow \begin{array}{l} 4x - 6y = 8 \\ 9x + 6y = -6 \\ \hline 13x \quad = 2 \end{array}$$

$$x = \frac{2}{13}$$

$$\therefore x = \frac{2}{13}\ ,\ y = \frac{16}{13}$$

$$(ii)\quad z+2iw=4+3i$$
$$2z+iw=3+4i$$

$$\Rightarrow \begin{array}{r} z+2iw=4+3i \\ 4z+2iw=6+8i \\ \hline 3z \qquad\quad =2+5i \end{array}$$

$$z=\frac{2}{3}+\frac{5}{3}i$$

$$\therefore \frac{2}{3}+\frac{5}{3}i+2iw=4+3i$$
$$2iw=\frac{10}{3}+\frac{4}{3}i$$
$$w=\frac{10}{6i}+\frac{4}{6}$$
$$=\frac{2}{3}-\frac{5}{3}i$$

$$w=\frac{2}{3}-\frac{5}{3}i \quad , \quad z=\frac{2}{3}+\frac{5}{3}i$$

Exercise 1A; 1 to 10 ace etc, 11bd, 13 to 20