Curve Sketching

Look for;

- points of discontinuity e.g. $y = \frac{1}{x}, x \neq 0, y \neq 0$
- asymptotes e.g. $y = x + \frac{1}{x}, x \neq 0, y \neq x$

On the curve $y = f(x)$

(1) stationary points occur when $f'(x) = 0$

(2) maximum turning point if $f'(x) = 0, f''(x) < 0$

(3) minimum turning point if $f'(x) = 0, f''(x) > 0$

(4) point of inflection if $f''(x) = 0$ and there is a change in concavity $[f'''(x) \neq 0]$

A curve is;

- increasing if $f'(x) > 0$
- decreasing if $f'(x) < 0$
- concave up if $f''(x) > 0$
- concave down if $f''(x) < 0$

e.g. Sketch the curve $y = x^3 - 6x^2 + 9x - 5$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\frac{d^2y}{dx^2} = 6x - 12$$

$$\frac{d^3y}{dx^3} = 6$$

Stationary points occur when $\frac{dy}{dx} = 0$

$$\text{i.e. } 3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1 \quad \text{or} \quad x = 3$$

when $x = 1, \frac{d^2y}{dx^2} = 6(1) - 12$

$$= -6 < 0$$

$\therefore$ $(1, -1)$ is a maximum turning point when $x = 3, \dfrac{d^2y}{dx^2} = 6(3) - 12$

$$= 6 > 0$$

$\therefore (3, -5)$ is a minimum turning point

Possible points of inflection occur when $\dfrac{d^2y}{dx^2} = 0$

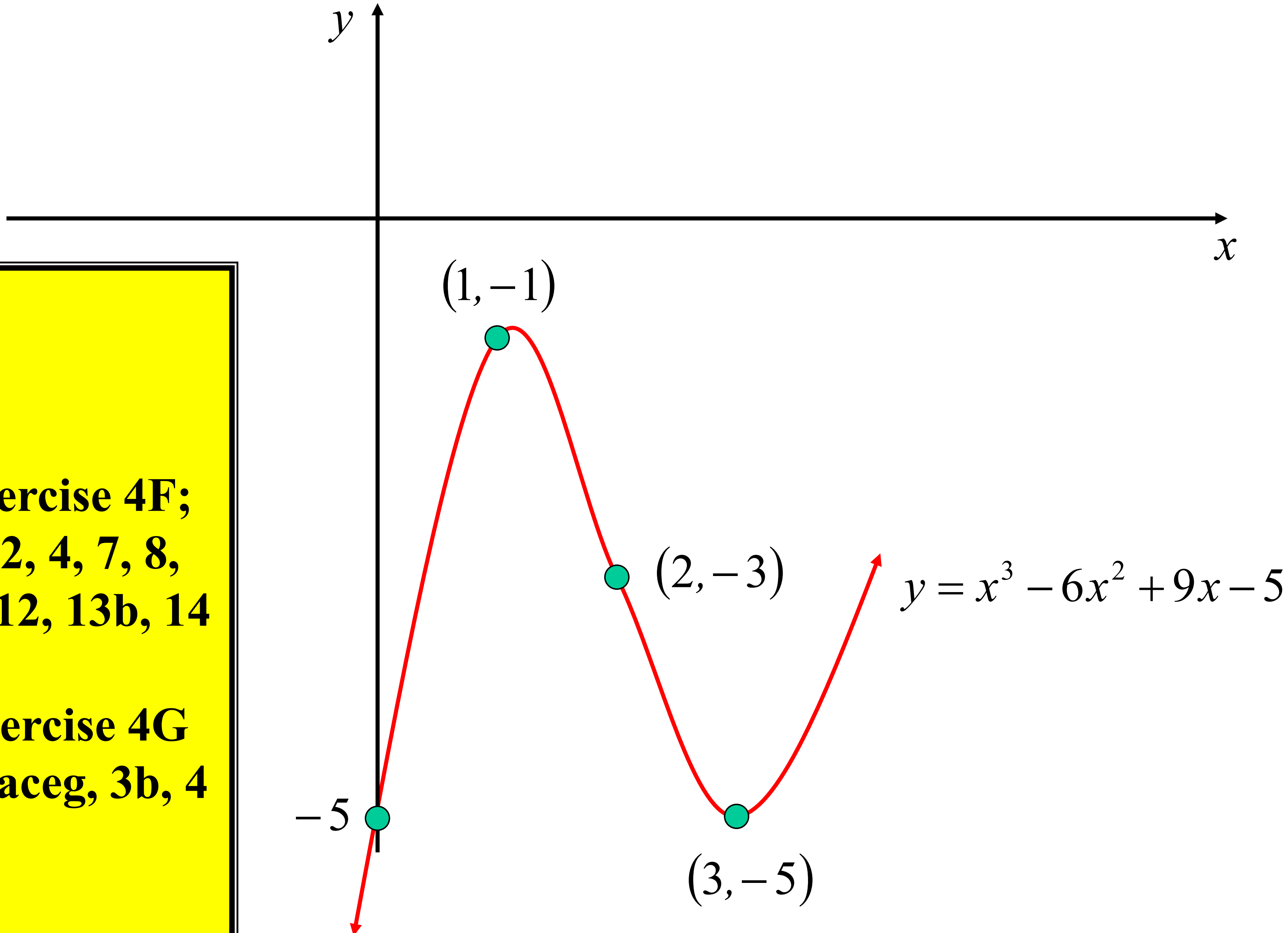
i.e. $6x - 12 = 0$

$$x = 2$$

when $x = 2, \dfrac{d^3y}{dx^3} = 6 \neq 0$

i.e. there is a change in concavity

$\therefore (2, -3)$ is a point of inflection



Exercise 4F;
1, 2, 4, 7, 8,
10, 12, 13b, 14

Exercise 4G
1, 2aceg, 3b, 4