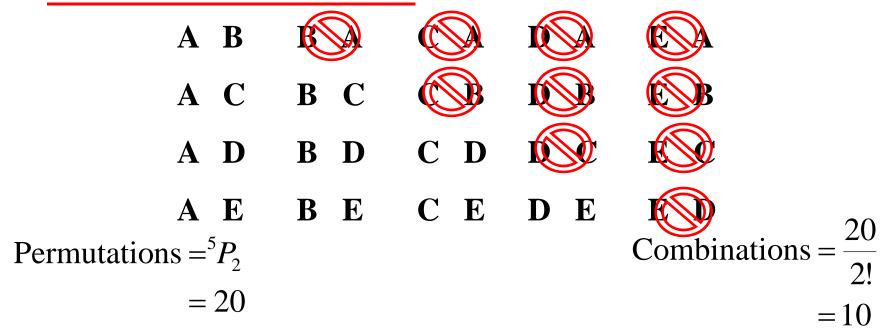
Combinations

A combination is a set of objects where the order that they are arranged is not important. i.e. a **selection**

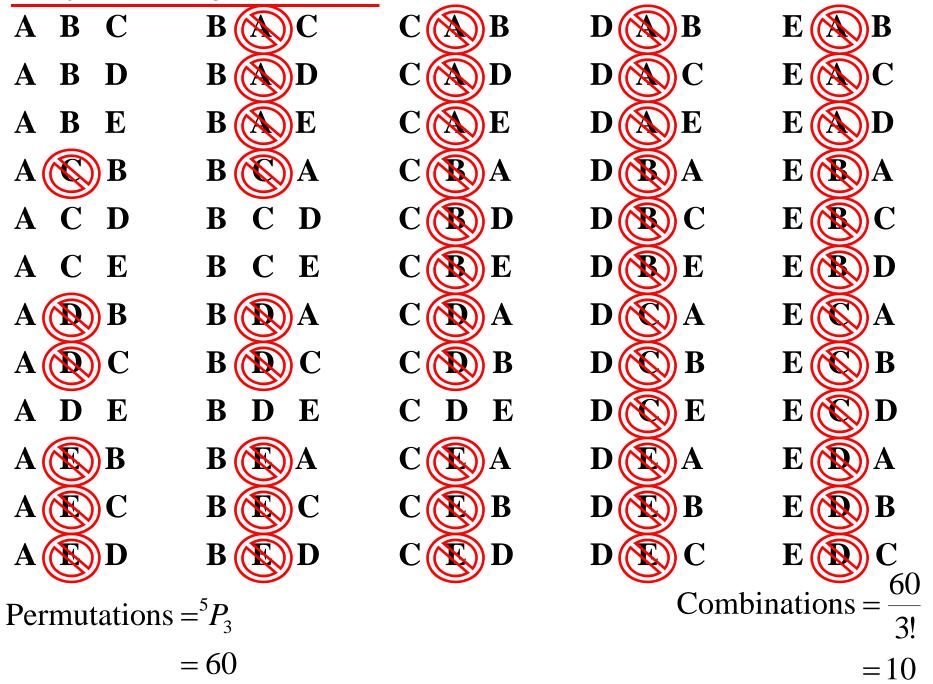
If we arrange objects in a line, and the order is not important then;

A B is the same selections as **B A**

e.g. 5 objects, arrange 2 of them







If we have *n* different objects, and we arrange *k* of them and are not concerned about the order;

Number of Arrangements =
$$\frac{{}^{n}P_{k}}{k!}$$

= $\frac{n!}{(n-k)!k!}$
= ${}^{n}C_{k}$

e.g. (*i*) How many ways can 6 numbers be chosen from 45 numbers?

Ways
$$= {}^{45}C_6$$

= 8145060

Note: at 70 cents per game, \$5 701 542 = amount of money you have to spend to guarantee a win in Lotto.

(*ii*) Committees of five people are to be obtained from a group of seven men and four women.

How many committees are possible if;

a) there are no restrictions?

$$Committees = {}^{11}C_5$$
$$= 462$$

With no restrictions, choose 5 people from 11, gender does not matter

b) the committee contains only males?

$$Committees = {}^{7}C_{5}$$
$$= 21$$

By restricting it to only males, there is only 7 people to choose from

c) the committee contains at least one woman?

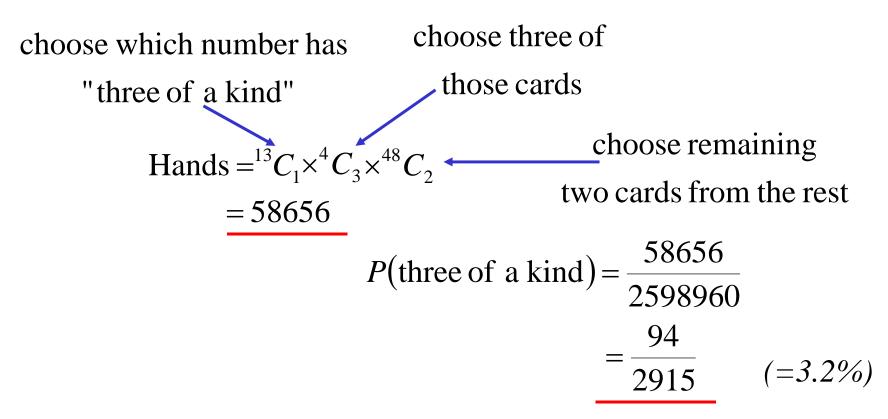
Committees = 462 - 21= 441

easier to work out male only and subtract from total number of committees (*iii*) A hand of five cards is dealt from a regular pack of fifty two cards.

a) What is the number of possible hands?

Hands $= {}^{52}C_5$ = 2598960

b) What is the probability of getting "three of a kind"?



2004 Extension 1 HSC Q2e)

A four person team is to be chosen at random from nine women and seven men.

(*i*) In how many ways can this team be chosen?

Teams $=$ ¹⁶ C_4	With no restrictions, choose 4 people
=1820	from 16, gender does not matter

(*ii*) What is the probability that the team will consist of four women?

Teams =
$${}^{9}C_{4}$$
By restricting it to only women, there is= 126only 9 people to choose from

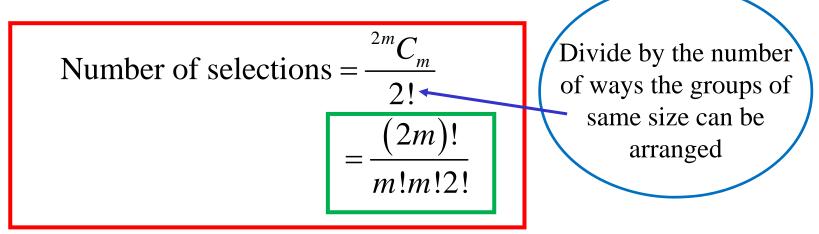
$$P(4 \text{ women team}) = \frac{126}{1820}$$
$$= \frac{9}{130}$$

Dividing into Groups

Case 1: dividing a group of (m + n) objects into two groups containing *m* objects and *n* objects



Case 2: dividing a group of (2*m*) objects into two groups each containing *m* objects



Case 3: dividing a group of (m + n + p) objects into two groups containing *m* objects, *n* objects and *p* objects respectively

Number of selections =
$${}^{m+n+p}C_m \times {}^{n+p}C_n$$

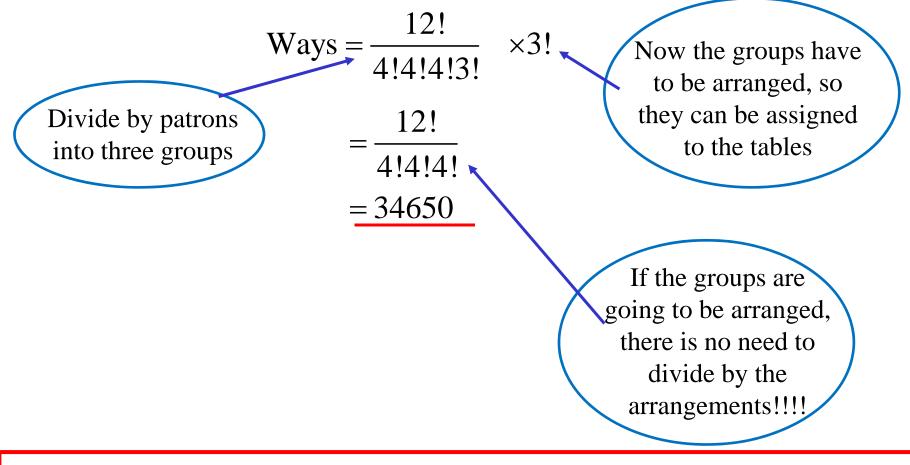
= $\frac{(m+n+p)!}{m!(n+p)!} \times \frac{(n+p)!}{n!p!}$
= $\frac{(m+n+p)!}{m!n!p!}$

e.g. (*i*) In how many ways can 15 students be divided into three equal groups?

Ways =
$$\frac{15!}{5!5!(3!)}$$

= 126126
Divide by three as
groups are of the
same size

(*ii*) A restaurant has 3 tables, each with 4 seats. 12 people arrive for dinner. How many ways can they assigned to the tables?



A permutation is a combination multiplied by the number of ways the objects can be arranged

i.e. select then arrange

Insertion Method

Useful when some objects are NOT allowed to be together

e.g. The letters of the word **BETWEEN** are arranged in a line. In how many ways can they be arranged if;

(*i*) All the **E**'s are separated?

Traditional method: work out the number of ways the E's can be placed

E E		Ε	E	E	E E	E E	3
					Ľ		
\mathbf{E}				Ε		E	1
	E		\mathbf{E}		E	E	2
	\mathbf{E}			E		E	1
		Ε		Ε		Ε	1
Ways $=10 \times 4!$							10
= 240							

Insertion method: place the unrestricted objects first, creating spots for the E's to be placed.

Once **B**, **T**, **W** and **N** have ben arranged there are now 5 spaces that the **E**'s can go (note: order not important as objects are the same)

Ways =
$$4! \times {}^{5}C_{3}$$

= 240

(*ii*) Exactly two of the **E**'s are together?

(note: this time order is important as objects are not the same **EE** and **E**)

Ways =
$$4! \times {}^5P_2$$

= 480



Useful when dividing large groups into smaller groups

e.g. Three pirates are sharing out the contents of a treasure chest containing forty-eight gold coins and two lead coins.

The first pirate takes out coins one at a time until a lead coin is taken.

- The second pirate then takes out coins one at a time until the second lead coin is taken.
- The third pirate then takes all of the remaining coins.

In how many ways can the coins be distributed?

The question is equivalent to how many ways can 2 L's and 48 G's be arranged. (The 2 L's act as separators of the 3 pirates)

Ways
$$=\frac{50!}{48!2!}$$

= 1225

2012 Extension 2 HSC Question 16 a) (ii)

In how many ways can 10 identical coins be allocated to 4 different boxes?

The question is equivalent to how many ways can 3 S's and 10C's be arranged. (The 3 S's act as separators of the 4 boxes)

Ways
$$=\frac{13!}{3!10!}$$

= 286

Note: 1% of the state got this correct!!!

2013 Extension 2 HSC Question 10

A hostel has four vacant rooms. Each room can accommodate a maximum of four people.

In how many ways can six people be accommodated in the four rooms?

Total Ways no restrictions $= 4^6$ Each person has a choice of 4 rooms =4096Less ways with 6 in a room = ${}^{4}C_{1}$ *Choose which of* the 4 rooms will = 4have the six people Less ways with 5 in a room = ${}^{4}C_{1} \times {}^{6}C_{5} \times {}^{3}C_{1}$ =72Choose which of the 4 rooms will have the five people, then choose Ways = 4096 - 4 - 72the five people to go in that room, = 4020then which of the remaining rooms will have one

Note: 15% of the state got this correct!!! It was multiple choice

Exercise 14E; 1, 3, 5, 7, 9, 11, 13, 14, 17, 19, 21, 22, 23c, 25a, 26, 28, 29