Multi-Stage Experiments

Independent events are events whose outcome has no affect on the other events

If $A, B, C, D,\ldots$ are independent events, then

$$P(ABCD...) = P(A)\times P(B)\times P(C)\times P(D)\times ...$$

e.g. In a game of Yahtzee, 5 dice are rolled. Find the probability of rolling exactly four 6's

$$P(\text{four 6's}) = P(6666\bar{6}) + P(666\bar{6}6) + P(66\bar{6}66) + P(6\bar{6}666) + P(\bar{6}6666)$$

$$= \frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6} + \frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}$$

$$+\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}$$

$$= \frac{5}{7776} + \frac{5}{7776} + \frac{5}{7776} + \frac{5}{7776} + \frac{5}{7776} = \frac{25}{7776}$$

OR

With independent events the probability will remain the same, regardless of the order

$$P(\text{four 6's}) = 5 \times P(6666\bar{6})$$
$$= 5 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$$
$$= \frac{25}{7776}$$

If the events are **not** independent, all different cases need to be considered.

e.g. Two letters of the word **LOTTO** are randomly arranged.

What is the probability that the arrangement is **TO** ?

As the choice of letters is not equally likely, it is **NOT** true to say

$$P(\text{TO}) = \frac{2}{5} \times \frac{2}{4} = \frac{1}{5}$$ ✗

Possibilities

LO OL

TO OT

OO TT

TL LT

$$P(\text{TO}) = \frac{1}{8}$$

Exercise 12E; 3, 5, 7, 9, 10, 11, 13, 15, 16