

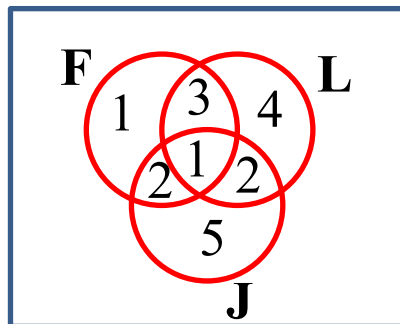
# Conditional Probability

The conditional probability of an event  $A$ , given that event  $B$  has already occurred is given by;

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{or} \quad P(A|B) = \frac{|A \cap B|}{|B|}$$

*Note: if  $P(A|B) = P(A)$  then  $A$  and  $B$  are independent events  
or  $P(A \cap B) = P(A) \times P(B)$*

e.g. (i) In a mixed language class, students study French, Latin and Japanese. The number of students who study each language are shown in the Venn diagram.



What is the probability that a student who studies Japanese also studies French?

$$P(\text{French} | \text{Japanese}) = \frac{3}{10}$$

(ii) Two boxes each contain four stones that differ only in colour.

Box 1 contains four black stones

Box 2 contains two black stones and two white stones

A box is chosen at random and one stone is randomly drawn from it

a) What is the probability that the randomly drawn stone is black

$$P(\text{black}) = \frac{1}{2} + \frac{1}{2} \times \frac{2}{4} \quad \text{OR} \quad P(\text{black}) = \frac{6}{8}$$
$$= \frac{3}{4} \quad \quad \quad = \frac{3}{4}$$

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b) It is not known from which box the stone has been drawn.

Given that the stone drawn is black, what is the probability that it was drawn from Box 1?

$$\begin{aligned}P(\text{Box 1} | \text{black}) &= \frac{P(\text{black and Box 1})}{P(\text{black})} \\ &= \frac{\frac{1}{2} \times 1}{\frac{3}{4}} \\ &= \frac{2}{3}\end{aligned}$$

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(iii) In a particular school 55% are male and 45% are female. Of the male students 13% say Monday is their favourite day, while 18% of the females say Monday is their favourite.

Find the probability that a student chosen at random is a male whose favourite day is Monday.

$$P(\text{Male}) = 0.55$$

$$P(\text{Monday}|\text{Male}) = 0.13$$

$$P(\text{Monday}|\text{Male}) = \frac{P(\text{Monday} \& \text{Male})}{P(\text{Male})}$$

$$0.13 = \frac{P(\text{Monday} \& \text{Male})}{0.55}$$

$$\begin{aligned} P(\text{Monday} \& \text{Male}) &= 0.13 \times 0.55 \\ &= \underline{0.0715} \end{aligned}$$

**(iv) 2022 Advanced HSC Question 15**

In a bag there are 3 six-sided dice. Two of the dice have faces marked 1, 2, 3, 4, 5, 6. The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

a) What is the probability that the die shows a 5?

$$\begin{aligned} P(5) &= \frac{2}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} \\ &= \underline{\underline{\frac{5}{18}}} \end{aligned}$$

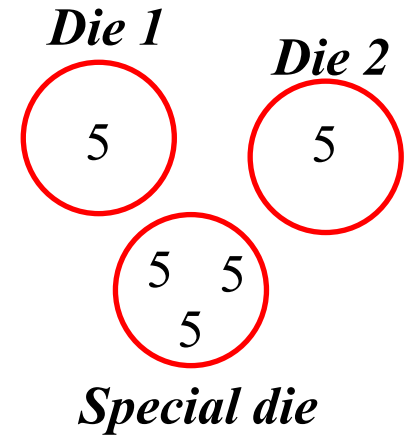
b) Given that the die shows a 5, what is the probability that it is the special die?

$$P(\text{special die} \mid 5) = \frac{P(\text{special die} \& 5)}{P(5)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{5}{18}}$$

$$= \underline{\underline{\frac{3}{5}}}$$

**OR**



$$P(\text{special die} \mid 5) = \underline{\underline{\frac{3}{5}}}$$

**Exercise 12G; 1ac, 2, 3, 5, 6ac, 7ac, 8ace, 10, 11, 13, 14, 15,  
17, 18, 19, 21, 22, 25**